COORDINATE GEOMETRY - III

Semester-I

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1. Prove that the locus of the points such that two of the three normals to the parabola $y^2 = 4ax$ from then coincide is $27ay^2 = 4(x - 2a)^3$. 5½

2. (i) Prove that sub-normal at any point of a parabola is constant and equal to half the latus rectum. 3

   (ii) Prove that the locus of the foot of the perpendicular from the focus on any tangent to a parabola is the tangent at that vertex. 2½

3. Show that the locus of the middle point of variable chord of the parabola $y^2 = 4ax$ such that the focal distance of the extremities are in the ratio of 2 : 1 is $9(y^2 - 2ax)^2 = 4a^2(2x - a)(4x + a)$. 5½

4. Find the locus of the point such that two of the normals drawn through it to the parabola $y^2 = 4ax$ are perpendicular to each other. 5½

Section - B

5. Prove that the locus of the feet of the perpendiculars from the foci on any tangent to an ellipse is the auxiliary circle and product of the perpendiculars is equal to the square of the semi minor axis. 5½

6. Find the combined equation of tangents on the ellipse $\frac{x^2}{4} + \frac{y^2}{3}$ from the point (–16, 9) and also find the angle between them. 5½

7. (i) Prove that four normals can be drawn to the hyperbola $axy = c^2$ from any given point. 2½

   (ii) Prove that the area of the triagle formed by any tangent tot he hyperbola $x^2 - y^2 = a^2$ and its asymptotes is constant. 3

8. Prove that the locus of foot of perpendicular from the centre of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ on any tangent is $(y^2 + x^2)^2 = a^2x^2 - b^2y^2$. 5½

Section - C

9. (i) For what value of k the equation $\frac{x^2}{9-k} + \frac{y^2}{5-k} = 1$ represents an ellipse ? 2

   (ii) Prove that the chord of contact of any point on the directrix of a parabola passses through the focus. 2

   (iii) Find the locus of mid points of the chords of the parabola $y^2 = 4ax$ which touch the circle $x^2 + y^2 = a^2$. 2

   (iv) Show that asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are the diagonals of the fundamental rectangle. 2

   (v) If e_1 and e_2 are accentricities of a hyperbola and conjugate hyperbola, then prove that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$. 2

   (vi) Find the length of the chord intercepted by the ellipse $x^2 + 2y^2 = 4$ on the normal at the point (2, 1) 2

   (vii) What is director circle ? Find the quation of the director circle of the ellipse $16x^2 + 9y^2 = 144$. 2