COORDINATE GEOMETRY - III

Semester - I

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1 The two tangents from a point P to the parabola $y^2 = 4ax$ make the angles α, β with x axis. Find the locus of P when.

(i) $\tan \alpha + \tan \beta$ is constant. (ii) $\tan^2 \alpha + \tan^2 \beta$ is constant. 5½

2. Prove that the locus of the points such that two of the three normals to the parabola $y^2 = 4ax$ from them coincide is $27ay^2 = 4(x - 2a)^3$. 5½

3. Show that segment of the tangent to a parabola cut off between the directrix and the curve subtends a right angle at the focus of the parabola. 5½

4. Prove that the semi-latus rectum of a parabola is the harmonic mean between the segments of a

focal chord. 5½

Section - B

5. (i) Prove that the normal at any point of an ellipse bisects the angle between the focal distances of the point. 3

(ii) Prove that the eccentric angles of the extremities of two conjugate diameters of an ellipse differ by a right angle. 2½

6. If A and B are the extremities of the conjugate diameters on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then show that the tangents at A and B meet on the elliplse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ and that of the locus of the middle point of AB is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$. 5½

7. A circle cuts the rectangular hyperbola $xy = 1$ in the points (x_r, y_r) where $r = 1, 2, 3, 4$. Prove that $x_1x_2x_3x_4 = y_1y_2y_3y_4 = 1$. 5½

8. Prove that the locus of the middle points of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$. 5½

Section - C

9. Do as directed :

(i) What is director circle ? Find equation of Director circle of the ellipse $9x^2 + 25y^2 = 225$.

(ii) Find the asymptotes to the hyperbola : $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$

(iii) Find the equation of the normal to the parabola $y^2 = 12x$ which is perpendicular to the line $x - 3y + 6 = 0$.

(iv) The normal at a point t_1 on the parabola $y^2 = 4ax$ meets it again at the point t_2. Prove that $t_2 = -t_1 - \frac{2}{t_1}$

(v) Find k so that $kx - y = 1$ is a normal to the conic $x^2 + y^2 = 1$.

(vi) Find the eccentricity of the hyperbola : $16(x - 1)^2 - 9(y - 2)^2 = 144$.

(vii) Find the curve whose parametric equations are $x = e^t + e^{-t}$, $y = e^t - e^{-t}$. 7×2=14