DIFFERENTIAL EQUATIONS - II

Semester - I

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 5.5 marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1 (a) Find the differential equation of the family of curves $(x - h)^2 + (y - k)^2 = r^2$, where h and k are arbitrary constants. 2.5

(b) Solve the differential equations $(D^4 + 2D^2 + 1)\, y = x^2 \cos x$, where $D = \frac{d}{dx}$.

2. (a) Solve $(D^5 - D)\, y = 12\, e^x + 8 \sin x - 2x$. 2.5

(b) Find the necessary and sufficient condition that the equation $Mdx + Ndy = 0$ (where M and N are the functions of x and y with the condition that M, N, $\frac{\partial M}{\partial y}$, $\frac{\partial N}{\partial x}$ are continuous functions of x and y) may be exact. 3

3. (a) Discuss the Method of Variation of Parameters. 2.5

(b) Solve $(x^2y^2 + xy + 1)y\, dx + (x^2y^2 - xy + 1)x\, dy = 0$. 3

4. (a) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 5(3x+2)\frac{dy}{dx} - 3y = x^2 + x + 1$. 2.5

(b) By the Method of Changing the Independent Variable, solve :

$$x^6\frac{d^2y}{dx^2}+3x^5\frac{dy}{dx}+a^2y=\frac{1}{x^2}.$$ 3

Section - B

5. (a) Use operator method to find the general solution of the linear system $\frac{dx}{dt}+\frac{dy}{dt}-x-3y=e^t$, $\frac{dx}{dt}+\frac{dy}{dt}+x=e^{3t}$. 2.5

(b) Solve in the power series about x = 2 the equation :

$$\frac{d^2y}{dx^2}+(x-1)\frac{dy}{dx}+y=0.$$ 3

6. (a) Find the general solution of the linear system :

$$\frac{dx}{dt}=5x-y,\ \frac{dy}{dt}=3x+y.$$ 2.5

(b) Solve in series $2x\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}-2y=0$, about x = 0. 3

7. (a) Show that $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$. 2.5

(b) Prove that $(1-2xK+K^2)^{-1/2}$ where $|x|\le 1$, $|K|<1$ is a generating function for Legendre polynomials $P_n(x)$. 3

8. (a) Prove that $J_n(-x)=(-1)^n J_n(x)$ if n is any integer. 2.5

(b) Prove that $\int_{-1}^{1}xP_{n-1}(x)P_n(x)dx=\frac{2n}{4n^2-1}$. 3

Section - C

9. Write in short :

(a) Define Integration Factor of a differential equations.

(b) Solve $(D^3+1)y=3+e^{-x}$.

(c) Using Wronskian, show that x, e^x, xe^x, $(2-3x)e^x$ are linearly dependent, for any real x.

(d) Define ordinary and singular points of a differential equation.

(e) Show that $J_n(x)$ is convergent for all x.

(f) Express $x^4+x^3+x^2+8x-3$ in terms of Legendre's polynomials.

(g) Show that $J_0^2+2\left(J_1^2+J_2^2+\dots\dots\dots\right)=1$. 2×7=14