DIFFERENTIAL EQUATIONS - II

Semester-I

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 5.5 marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1. (a) Find the differential equation of all ellipses centered at the origin and foci on x-axis. 2.5

   (b) Solve the differential equation $\frac{d^2y}{dx^2} - y = x^2 \cos x$. 3

2. (a) Solve $(D^5 - D)y = e^x + 8 \cos x - 12x$. 2.5

   (b) Find the necessary and sufficient condition that the equation $Mdx + Ndy = 0$ (where M and N are the function of x and y with the condition that M, N, $\frac{\partial M}{\partial y}, \frac{\partial N}{\partial x}$ are continuous functions of x and y) may be exact. 3

3. (a) Discuss the Method of variation of Parameters. 2.5

(b) Solve the differential equation $\left(xy^2 - e^{1/x^2}\right)dx - x^2y\,dy = 0$.

4. (a) Solve : 3

$$(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=3x^2+4x+1.$$ 2.5

(b) By the Method of Changing the Independent Variable, solve :

$$\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-y\sin^2 x=0.$$ 3

Section - B

5. (a) Use operator method to find the general solution of the linear system :

$$2\frac{dx}{dt}+\frac{dy}{dt}+x+5y=4t,\quad \frac{dx}{dt}+\frac{dy}{dt}+2x+2y=2.$$ 2.5

(b) Show in the power series about x = 1 the equation :

$$\frac{d^2y}{dx^2}+(x-1)^2\frac{dy}{dx}-4(x-1)y=0.$$ 3

6. (a) Find the general solution of the linear system :

$$\frac{dx}{dt}=5x-2y,\quad \frac{dy}{dt}=4x-y.$$ 2.5

(b) Solve in series $\left(1+x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$, about x = 0, where n is positive integer. 3

7. (a) Show that $J_{\frac{-3}{2}}(x)=\sqrt{\frac{2}{\pi x}}\left(-\frac{\cos x}{x}-\sin x\right)$. 2.5

(b) Show that $\int_{-1}^{1}x^K P_n(x)\,dx=0$, where K is an integer less than n. 3

8. (a) Prove that $\int_0^{\frac{\pi}{2}}J_0(x\cos\phi)\cos\phi\,d\phi=\frac{\sin x}{x}$. 2.5

(b) Prove that $\int_{-1}^{1}\left(P_n(x)\right)^2dx=\frac{2}{2n+1}$; if m = n. 3

Section - C

9. (a) Solve the Beroulli's equation $\frac{dy}{dx}+Py=Qy^n$. Where P and Q are functions of x.

(b) Solve $\frac{d^4y}{dx^4}-\frac{d^2y}{dx}=2$.

(c) Using Wronskian, show that sin x, cos x, 3 sin x – 4 cos x are linearly dependent, for any real x.

(d) Define ordinary and singular points of a differential equation.

(e) Show that :

$$J_1(x)=\frac{x}{2}-\frac{1}{\angle 1\angle 2}\left(\frac{x}{2}\right)^3+\frac{1}{\angle 2\angle 3}\left(\frac{x}{2}\right)^5-\frac{1}{\angle 3\angle 4}\left(\frac{x}{2}\right)^7+\ldots$$

(f) Express $3x^3 + x^2 + 8x - 3$ in terms of Legender's polynomials.

(g) State Rodrigue formula.

2×7=14.