ALGEBRA - I

Paper - IV
Semester - II

Time : Three Hours] [Maximum Marks : 36

Note: Attempt *two* questions each from Section A and B carrying 5.5 marks each, and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1. (a) For what value of μ the the equations

$$x + y + z = 1,$$
$$x + 2y + 4z = \mu,$$
$$x + 4y + 10z = \mu^2$$

have a solution ? Solve them completely in each case. 3

(b) Prove that characteristic roots of a unitary matrix are of unit modulus. 2½

2. (a) Define Similar matrices, and prove that similar matrices have same characteristic polynomial and hence eigen values. 3

(b) Find modal matrix of the matrix $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$. 2½

3. (a) Determine weather the follwoing matrices have same column space or not :

$$\begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 9 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 17 \end{bmatrix}.$$ 2½

(b) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, find using Cayley Hamilton theorem matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. 3

4. (a) If $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$, find minimal polynomial of A. 2½

(b) Find matrix P & Q. Such tha PAQ is in the normal form where $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 1 \end{bmatrix}$. 3

Section - B

5. (a) If product of two roots of $x^4 + px^3 + 7q + rx + x = 0$ is equal to the product of other two then show that $r^2 = p^2s$. 3

(b) If α, β, γ are roots of $x^3 - x^2 + 1 = 0$, form an equation whose roots are $\alpha + \beta^2 + \gamma^2 + \alpha^2 + \gamma^2, \gamma + \beta^2 + \alpha^2$. 2½

6. (a) Find an equation whose roots are squared differences of the roots of the cubic $x^3 - 6x + 4\sqrt{2} = 0$. 2½

(b) Solve the following equation by Descarte's method : $2x^4 + 6x^3 - 3x^2 + 2 = 0$ 3

7. (a) Express $\cos^2\theta \sin^6\theta$ in series of sine of multiple of θ. 2½

(b) Obtain an equation whose roots are $\sec^2\frac{\pi}{7}, \sec^2\frac{3\pi}{7}, \sec^2\frac{5\pi}{7}$. 3

8. (a) Prove that if $\tan\log(x + iy) = a + ib$ and $a^2 + b^2 \neq 1$ then prove that $\tan\log(x^2 + y^2) = \frac{2a}{1 - a^2 - b^2}$. 2½

(b) Sum the series $\cos^2\theta - \frac{1}{3}\cos^3\theta\cos 3\theta + \frac{1}{5}\cos^5\theta\cos 5\theta + ...\infty$ 3

Section - C

9. Attempt all the following :

(a) Show that if A is hermitian then so is $A + A^t$.

(b) State Decrate's Rule of signs.

(c) Define Diagonalizable matrix.

(d) Prove that $(-i)^{-1} = e^{(4x-1)\frac{\pi}{2}}$.

(e) Solve the equation $x^4 + 1 = 0$.

(f) Separate $e^{(6+5i)^2}$ into real and imaginary parts.

(g) Using Cayley-Hamilton theorem, find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ (7×2=14)