ANALYTIC GEOMETRY – VI

Time : Three Hours] [Maximum Marks : 36

Note : Attempt two questions each from Section A and B carrying 5.5 marks each, and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

SECTION - A

I. (a) Prove that if a plane has the intercepts a, b, c and is at a distance p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$. (2.5)

(b) Find the area of the triangle included between the plane $2x - 3y + 4z = 12$ and the coordinate planes. (3)

II. (a) Prove that $2x^2 - 2y^2 + 4z^2 + 6zx + 2yz + 3xy = 0$ represents a pair of planes, and find the acute angle between them.

(b) Prove that the join of (1, 2, 3), (2, 1, 4) is normal to the plane through (5, –1, –9), (1, 0, –4), (–1, 2, 0). (3)

III. (a) Find the equation of the projection of the line $\frac{x-1}{2}=\frac{y}{-5}=\frac{z}{3}$ on the plane $5x - 4y - z = 0$. (2.5)

(b) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{5}, \frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar, point and the equation of the plane in which they lie. (3)

IV. (a) Show that the S.D. between lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}, \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ also find its equations and the points in which it meets the given lines. (2.5)

(b) Prove that planes $2x - y + z = 0$, $5x + 7y + 2z = 0$ and $3x + 4y - 2z + 3 = 0$ meet in a point. Find the coordinates of their point of intersection. (3)

SECTION - B

V. (a) Find equation of sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point P(1, –2, 1) and also cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$. (2.5)

(b) State and prove the condition of orthogonality of two spheres. (3)

VI. (a) Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x + y + z - 3 = 0$ as a great circle. Also, determine its radius and centre. (2.5)

(b) Find the equation of the right circular cone whose vertex is at the origin, whose axis is the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and which has a vertical angle of 60°. (3)

VII. (a) Find the equation of the cone formed by rotating the line $z = 0$, $y = 2x$ about the axis. (2.5)

(b) Show that $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$ represents a cone. Also find the coordinates of this cone. (3)

VIII. (a) Find the equation of cone whose vertex is (2, –3, 1) and whose guiding curve is $4x^2 + y^2 = 1$, $z = 0$ (2.5)

(b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 0$. (3)

SECTION-C

IX. Attempt all the following :

(a) Find power of the point (2, –1, 4) w.r.t. the sphere $2x^2 + 2y^2 + 2z^2 - 3x + 4y + 5z - 11 = 0$.

(b) Find the equation of the sphere whose centre is the point (1, 2, 3) and which touches the plane $3x + 2y + z + 4 = 0$.

(c) Find the equation of the cone with vertex at the origii and which passes through the curve

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given by $x^2 + y^2 + z^2 - x - 1 = 0$ and $x^2 + y^2 + z^2 + y - 2 = 0$

(d) Find the equation of the plane through the points (3, –1, 2), (1, –1, –3) and (4, –3, 1).

(e) Find angles between the planes $2x - y + z = 6$ and $x + y + 2z = 7$.

(f) Find area of triangle included between the plane $2x - 3y + 4z = 12$ and the coordinate planes.

(g) Find the point where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$ meet the plane $3x + 4y + 2z = 7$.

(7×2=14)

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