ANALYTIC GEOMETRY - VI

Semester - II

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : Attempt *two* questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1 (a) Prove that if a plane has the intersepts a,b, c and is at a distance p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$ 2½

(b) A variable plane is at a constant distanced p from the origin and meets axis in A, B and C respectively, then show that locus of the centroid of the triangle ABC is $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{9}{p^2}$. 3

2. (a) Find the equations to perpendicular from the origin to the line $x + 2y + 3z + 4 = 0$, $2x + 3y + 4z + 5 = 0$. Find also the co-ordinates of the foot of the perpendicular. 2½

(b) Find the distance of the point (2, 3, 4) from the plane $3x + 2y + 2z + 5 = 0$ measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$. 3

3. (a) Find the equation of the projection of the line $\frac{x-1}{2}=\frac{y}{-5}=\frac{z}{3}$ on the plane $5x - 4y - z = 5$. 2.5

(b) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$, $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-3}{3}$ are coplanar, point and the equation of the plane in which they lie. 3

4. (a) Show that the S.D. between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$, $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$, and that its equation are $11x + 2y + 7z + 6 = 0$, and $7x + y - 5z = 7 = 0$. 2.5

(b) Prove that planes $2x - y + z = 0$, $5x + 7y + 2z = 0$ and $3x + 4y - 2z + 3 = 0$ meet in a point. Find the coordinates of their of intersection. 3

Section - B

5. (a) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its radius as small as possible. 2.5

(b) Find the centres of two spheres, which touch the plane $4x + 3y = 37$ at the point (8, 5, 4) and the sphere $x^2 + y^2 + z^2 = 1$. 3

6. (a) Show that locus of point from which equal tangents may be drawn to the spheres
$x^2 + y^2 + z^2 = 0$,
$x^2 + y^2 + z^2 + 2x - 2y + 2z - 1 = 0$,

$x^2 + y^2 + z^2 - x + 4y - 6z - 2 = 0$

is the straight line $\frac{x-1}{2} = \frac{y-2}{5} = \frac{z-1}{3}$. 2½

(b) A sphere of constant radius k passes through the origin and meets the axis in A, B and C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$. 3

7. (a) Find the equation to the right circular cone whose vertex is P(2, –3, 5), axis PQ which makes equal angles with the axes and which passes through A(1, –2, 3). 3

(b) Find equation of cone whose vertex is (2, –3, 1) and whose guiding curve is $4x^2 + y^2 = 1$, $z = 0$. 2.5

8. (a) Prove that the equation $\sqrt{fx} + \sqrt{gy} + \sqrt{hz} = 0$ represents a cone which touches the coordinte planes and the equation of reciprocal cone is $fyx + gzx + hxy = 0$. 2½

(b) Find the equation to the lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. Also find angle between them. 3

Section - C

9. Attempt all the following :

(a) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 1$, $x + 2y + 3z = 4$ nd the origin.

(b) Find the value of k such that $x^2 + y^2 + z^2 + 2x - 4y + 6z + k$ represents a sphere of radius 5.

(c) Find the equation of the cone which passes through the co-ordinates axes and lines $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and $\frac{x}{3} = \frac{y}{-1} = \frac{z}{1}$.

(d) Find the equation of the plane through the points (3, –1, 2), (1, –1, –3) and (4, –3, 1).

(e) Find area of triangle included between the plane $2x - 3y + 4z = 12$ and the coordinate planes.

(f) Find the point where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$ meets the plane $3x + 4y + 2z = 7$.

(g) Find the coordinate of the image of origin in the plane $2x + 3y - 4z = -1$. (2×7=14)