DIFFERENTIAL EQUATIONS - I

(Syllabus December 2013)

Time Allowede : Three Hours] [Maximum Marks : 40

Note : Attempt candidates are required to attempt *two* questions each from Section A and B carrying 8 marks each and the entire Section C consisting of 8 short answer type questions carrying 1 mark each.

Section - A

1. (i) Solve the given differential equations : $\left(y^3 - 2yx^2\right)dx + \left(2xy^2 - x^3\right)dy = 0$ 4

(ii) Solve the differential equation : $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y.$ 4

2. Solve the differential equations : $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3.$ 8

3. (i) Using method of variation of parameter, solve $(D^2 + 4)y = 4 \sec^2 2x$. 4

(ii) Define Legender's Linear equation and hence solve it. 4

4. Solve the differential equation : $\frac{d^2y}{dx^2} - y = x^2 \cos x$. 8

Section - B

5. (i) Using differential operator method, find general solution of :

$$\frac{dx}{dt} + \frac{dy}{dt} - x - 3y = e^t, \quad \frac{dx}{dt} + \frac{dy}{dt} + x = e^{3t}.$$ 4

(ii) Show that $P_n(x) = F\left(-n, n+1, 1; \frac{1-x}{2}\right)$,

where $P_n(x)$ is Legendre's function of order n. 4

6. (i) Using Recurrence relations for Bessel's function, prove that :

$$J_{-\frac{5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{3-x^2}{x^2}\cos x + \frac{3}{x}\sin x\right).$$ 4

(ii) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(9x^2 - \frac{1}{9}\right)y = 0$. 4

7. (i) Prove that $\int_{-1}^{1} x P_{n-1}(x) P_n(x)\,dx = \frac{2n}{4n^2 - 1}$. 4

(ii) Show that $J_0(x) = g\frac{1}{\pi}\int_0^{\pi} \cos(x\cos\phi)\,d\phi$ satisfies Bessel's equation of order zero. 4

8. Solve $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = e^x$ in series where $y(0) = 10$, $y'(0) = 20$. Hence find polynomial approximation of fifth degree to the solution. 8

Section - C

9. Do as directed :

(i) Find the differential equation of all circles having the centre on x-axis.

(ii) Solve $(D^2 + 1)^3 (D^2 + D + 1)^2 y = 0$

(iii) Find particular integral of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} = e^x$.

(iv) Show by Wornskian that the functions :

$1, 1 + x, 1 + x + x^2 + x^3$, where $x = \frac{-1}{3}$ are linearly independent over all reals.

(v) Define Radius of Convergence of a Power series.

(vi) Express $x^3 - x^2 + 4x - 6$ in term of Legendre's Polynomials.

(vii) Show that for $-1 < x < 1$, $F(1, \alpha, \alpha; x) = (1 - x)^{-1}$.

(viii) Show that $P_n(-x) = (-1)^n P_n(x)$. 8×1=8