PARTIAL DIFFERENTIAL EQUATION - V

Time : Three Hours] [Maximum Marks : 36

Note : Attempt five questions. Select two questions each from Section A and B and Q. No. IX of Section-C.

SECTION-A

I. (a) Solve by Lagrange's method $(y + z)p + (z + x)q = x + y$. 3

(b) Solve $z^2(p^2 + q^2 + 1) = a^2$. 2.5

II. (a) Solve $(y - z)p + (z - x)q = (x - y)$. 3

(b) Show that all the characteristic curves of the partial differential equation $(2x + u)u_x + (2y + u)u_y = u$ through the point (1, 1) are given by the same straight line $x - y = 0$. 2.5

III. (a) Solve using Charpit's method $q + xp = p^2$. 3

(b) Find a series solution of the following differential equation : $4y'' + 9xy = 0$. 2.5

IV. (a) Find the general solution of the partial differential equation $u_x^2 x + u_y^2 y - u = 0$. 3

(b) Solve $p^2(1 - x^2) - q^2(4 - y^2) = 0$. 2.5

SECTION - B

V. (a) Solve the partial differential equation $u_{tt} - c^2 u_{xx} = e^{-x} \sin t$. 3

(b) State one-dimensional Heat flow equation and find its solution. 2.5

VI. (a) Solve the equation $z_{xx} - 2z_x + z_y = 0$ by method of separation of variables. 3

(b) Solve $u_{xx} + y_{xy} - 6 = y \cos x$. 2.5

VII. (a) Solve $(D - 3D' - 2^{xy})^3 z = 6e^{2x} \sin(3x + y)$ 3

(b) Solve by method of separation $uxt = e^{-t} \cos x$ given that $u = 0$ when $t = 0$ and $u_t = 0$ when $x = 0$. 2.5

VIII. (a) Solve $u_{xx} - u_{xy} = \sin x \cos 2y$ 3

(b) Solve $x^{xx}r + 2^{xy}xys + y^2t = 0$. 2.5

SECTION-C

IX. (a) Examine whether the following partial differential equation is hyperbolic, parabolic or elliptic : $u_{xx} + xu_{yy} + 4 = 0$

(b) Find the general solution of $2u_x - 3u_y = \cos x$. Form the partial differential equation from $z = xf_1(x + t) + f_2(x + t)$.

(c) Solve $\sqrt{p} + \sqrt{q} = 2x$.

(d) Solve $2p + 3q = 1$.

(e) Classify the PDE $zp + pq = 0$.

(f) Solve $u_{xy} = xy$. xy.

(g) State and give the solution of Clairaut's equation in PDE. (7×2=14)