

VIBRATIONS AND WAVES - I
Paper - A, Semester - II

Time Allowed : Three Hours]

[Maximum Marks : 70

Note : (1) Candidates are required to attempt *two* questions each from Section A and B. Section C is compulsory.
All questions carry equal marks.
(2) Use of non programmable calculator is allowed.

Section : A

1. What is a compound pendulum ? Derive an expression for its time period. What is the condition for time period to be minimum ? 8
2. (a) Discuss the composition of two SHMs perpendicular to each other periods are in the ratio 1 : 2. 5
(b) The displacement of a simple harmonic oscillator is given by $x = a \sin(\omega t + \phi)$ if the oscillator started at time $t = 0$ from a position x and with a velocity v_0 show that : $\tan \phi = \frac{\omega x}{v_0}$ and $a = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$. 3
3. (a) Prove that the average kinetic energy of harmonic oscillator is equal to its average potential energy and each to half the total energy. 5
(b) A particle of mass 1m moved in a potential energy will be given by $U = 2x^2 + 4x = 4$. Find :
(i) the force constant 3
(ii) the frequency of oscillation. 4
4. (a) Define logarithmic decrement of a damped oscillator. Deduce the expression for it. 4
(b) Show that the ratio of the energy lost per cycle to the energy stored in a damped oscillator is $2\pi/Q$, where Q is the quality factor. 4

Section : B

5. Write down the equation of motion of forced oscillator being driven by an alternating force $F = F_0 \cos \omega t$. Explain the transient and steady state behaviour of forced oscillator. 8
6. (a) Derive an expression for the average power supplied by the driving force and the average power dissipated by the damping force or a forced oscillator. Show that they are equal. 5
(b) Find the frequency of a circuit containing inductance of $5 \times 10^{-2}\text{ H}$ and a capacitance of $5 \times 10^{-10}\text{ F}$. Find the wavelength of the radio wave to which it will respond. 3
7. (a) Show that in the resonant LCR circuit, the maximum potential drop across an inductor occurs at a frequency: $\omega = \frac{\omega_0}{\sqrt{1 - 2Q^2}}$ 3
(b) Show that the band width of the resonance absorption curve defines the phase angle $\tan \phi = \pm 1$. 3
8. (a) What is sharpness of Resonance curve ? Explain the effect of damping on sharpness of resonance. 3
(b) A r.m.s. voltage of 200 volts is applied to a series LCR circuit, having $R = 100\text{ ohm}$, $L = 10\text{ mH}$ and $C = 1\mu\text{F}$. Calculate :
(i) the natural frequency (ii) current resonance
(iii) Q-value of the circuit at resonance. 3

Section : C

9. Write a brief note on the following :

- (i) Explain how the interaction of inertia and elasticity produces SHM.
- (ii) Distinguish between free and forced oscillations.
- (iii) What is the physical significance of the mechanical impedance of forced oscillator?
- (iv) What is absorption resonance curve? Define absorption band width.
- (v) Is energy stored in forced oscillator? Explain.
- (vi) Are all periodic motion simple harmonic? Is the reverse true?
- (vii) Define quality factor of a damped oscillator.
- (viii) Examine whether the discharge of capacitor of $1 \mu\text{F}$ is oscillatory or not?

1×8=8