(vii) Define Relative velocity. (viii) Define Direct Impact.

ADVANCED CALCULUS–II

Semester - III

Time Allowed : Three Hours] [Maximum Marks : 50

Note : The candidates are required to attempt *five* questions in all, selecting *two* questions from Sections A and B carrying 10 marks each and Section C consiting of 10 short answer type questions carrying 1 mark each. Section C is compulsory.

Section : A

1. (a) Prove that Monotonic increasing sequence $\{x_n\}$ which is bounded above converges to its least upper bound.
   (b) Prove that if $\{x_n\}$ is Cauchy sequence then it is bounded also. 5+5
2. (a) A sequence $\{x_n\}$ converges to l if every subsequence of $\{x_n\}$ converges to l.
   (b) Test the convergence of series $\sum \frac{1^2.3^2.....(2n-1)^2}{2^2.4^2.....(2n)^2} x^{n-1}, x > 0.$ 5+5
3. (a) Prove that $\frac{1}{2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^4}+.....$ is convergent. Explain how Root Test is stonger than Ratio Test.
   (b) Let $\{x_n\}$ be sequence of real. Prove that if $\{t_n\}$ is a Cauchy sequence then so is $\{s_n\}$, where $s_n = x_1 + x_2 = \text{-------} + x_n$ and $t_n = |x_1| + |x_2|^n + \text{---------} + |x_n|$. 5+5
4. State and Prove D' A lembert's Ratio Test. 10

Section : B

5. (a) Discuss the continuity of function : $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} ; & (x,y) \neq (0,0) \\ 0 ; & (x,y) = (0,0) \end{cases}$ at (0, 0).
   (b) Prove that sufficient condition that a function f be continuous t (a, b) is tha one of the partial derivatives exist and is bounded in neighbourhood of (a, b) and hat the other exists as (a, b). 5+5
6. (a) Expand $e^x \sin y$ in Taylor's Series in neighbourhood of (0, 0).
   (b) Prove that a set is closed iff its complement is open. 5
7. (a) Prove that Cartesian product of two countable sets is countable.
   (b) Give an example to show tha a function maybe continuous and possess partial derivatives at a point and still may not be differentiable there at. 5+5
8. State and prove Heine Borel Theorem in IR. 10

Section : C

9. Attempt in brief :
   (a) Check the convergence of $3-2+\frac{4}{3}-\frac{8}{9}+\text{-------}$.
   (b) Prove that sequence cannot converge to more than one limit.
   (c) If $|x_n| < |y_n|$ $\forall n$ and $y_n \to 0$. Prove that $x_n \to 0$.
   (d) Prove that every Absolute Convergent Series is convergent.
   (e) What is a Homogeneous Function ? Give an example.
   (f) State Young's Theorem.
   (g) Define Closure of Set.

(h) What do you mean by Equivalent Sets ?

(i) If $z = x^3 - xy + y^3$, $x = r\cos\theta$, $y = r\sin\theta$. Find $\frac{\partial z}{\partial r}, \frac{\partial z}{\partial 0}$.

(j) Show that $u = \sin x + \sin y$, $v = \sin(x + y)$ are not functionally dependent. 10×1=10