ANALYSIS – II

Paper-V : Semester-IV

Time Allowed : Three Hours **Maximum Marks : 36**

Note : Attempt two questions each from Section A and B carrying 5½ marks each. Section C consisting of 8 short answer type questions carrying 14 marks in all is compulsory.

SECTION-A

I. (a) Show that the sequence $\{S_n\}$, where

$$S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\dots + \frac{1}{(n-1)!}$$ is convergent. 3

(b) Test the convergence of the series whose general term is $\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ 2½

II. (a) Show that if $b > 0$, the series

$$x + \frac{a-b}{2!}x^2 + \frac{(a-b)(a-2b)x^3}{3!} + \dots\dots$$

converges absolutely for $|x| < b^{-1}$. 3

(b) Show that

$$\lim_{n\to\infty} [1 + 2^{1/2} + 3^{1/3} + \ldots\ldots + n^{1/n}] = 1$$

2½

III. (a) Show that the series.

$$\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \ldots\ldots$$

converges for $p > 0$. 3

(b) Does the divergence of $\sum |u_n|$ imply the divergence of $\sum u_n$? Justify. 2½

IV. Discuss the convergence and divergence of the following series :

(i) $\sum \frac{r^n}{n^n}, r > 0$ 1½

(ii) $\frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \ldots\ldots\ldots\ldots\ldots$ 2

(iii) $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \ldots\ldots\ldots\ldots$ 2

SECTION–B

V. Show that $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots\ldots\ldots$ 5½

VI. Show that the series

$$\frac{1}{a} - \frac{2a}{a^2 - 1} \cos\theta + \frac{2a}{a^2 - 2^2} \cos 2\theta + \ldots\ldots\ldots$$

is uniformly convergent with respect to θ, in any finite interval. (5½)

VII. Prove that if a power series $\sum a_n x^n$ converges for $x = x_0$, then it is absolutely convergent for every $x = x_1$, with $|x_1| < |x_0|$. 5½

VIII. Show that the sequence $\{f_n\}$, where $f_n(x) = nx(1-x^2)^n$, converges but not uniformly to f, where $f(x) = 0$, for $0 \le x \le 1$ and that $\lim_{n\to\infty} \int_0^1 f_n\, dx \ne \int_0^1 f\, dx$. 5½

SECTION–C
(Compulsory Question)

IX. Attempt all the following :

(a) Find the interval of absolute convergence for the series $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$. 2

(b) State any two properties of functions expressible as power series. 2

(c) Define Power series and its radius of convergence. 2

(d) State Weierstrass M-test. 1

(e) Does every bounded sequence converge ? Justify. 2

(f) Find $\lim_{n\to\infty} \frac{1}{n}((m+1)(m+2)(m+3)\ldots\ldots. (m+n))^{1/n}$. 2

(g) State and prove the necessary condition for the convergence of an infinite series $\sum u_n$. 2

(h) State Liebnitz's test. 1