ANALYSIS–II

Paper - V Semester - IV

Time Allowed : Three Hours] [Maximum Marks : 40

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 8 marks each and the entire Section C consisting of 8 short answer type questions carrying 1 marks each.

Section : A

1. (a) Prove that $\sum(-1)^n \frac{x^3+n^2}{n^3}$ converges uniformly on every bounded subset or R. 3

(b) Prove that series $\sum u_n(x)v_n(x)$ is uniformly convegent on [a. b] if :

(i) $v_n(x)$ is a positive monotonic decreasing sequence converging uniformly to zero for a ≤ x^n ≤ b.

(ii) $|f_n(x)| = \left|\sum_{r=1}^{\infty} U_r(x)\right| < K$ for every value of x in [a, b] and all integral value of n, where K is a fixed number independent of x. 5

2. (a) Test for the term by term integratin, the series : $\sum_{n=0}^{\infty}(-1)^n x^n, 0 < x < 1$ 4

(b) Test for uniform convergence and continuity of the sum function of the series $f_n(x) = \frac{1}{1+nx}$ $0 \le x \le 1$. 4

3. (a) Show that the series $\sum \frac{1}{n^2+n^4x^2}$ is uniformly convergent for all real values of x and that it can be differentiated term by term.

(b) Show that the function represented by $\sum_{n=1}^{\infty}\frac{\sin nx}{n^3}$ is differentiable for every x and its derivative is $\sum_{1}^{\infty}\frac{\cos nx}{n^2}$. 4

4. (a) Find the radius of convergence of the p-series : $x + \frac{x^2}{2^2} + \frac{\lfloor 2}{3^3}x^3 + \frac{\lfloor 3}{4^4}x^4 + \ldots\ldots\ldots$ 4

(b) Show that $\tan^{-1} x - \frac{x^3}{3} + \frac{x^2}{5} - \frac{x^7}{7} + \ldots\ldots \; -1 \le x \le 1$. 4

Section : B

5. (a) Show that inverse point of the point 'a' w.r.t. circle |z–c| = R is the point $c + \frac{R^2}{\bar{a}-\bar{c}}$. 4

(b) If z = x + iy ; x, y ∈ R and $w = \frac{1-iz}{z-i}$, show that |w| = 1 ⇒ z is purely real. 4

6. (a) The complex number z number the conditions |z – 25 i| ≤ 15, find the number having the least positive argument.

(b) Show that q value of (cos θ + i sin θ) p/d from a G.P, whose sum is zero; p and q being integers prime to each other. 4

7. (a) Prove that the function u = x³ – 3x² – 3y² + 1 satisfies Laplace's equation and determine the corresponding analytic function. 4

(b) For what values of z do the function ω defined by equation : z = sin u cos h v + i cos u sin h v, ω = u + i v ceases to be anlytic ? 4

8. (a) Find the image of the circle |z – 2| = 2 under the modium Transformation $w = \frac{z}{z+1}$. 4

(b) Find the condition that the transformation $W = \frac{az+b}{cz+d}$ transforms the unit circle in W–plane into the straight line in z–plane. 4

Section : C

9. Do as directed :

(i) Write Cauchy's driterion for uniform convergence of series of function $\sum f_n$.

(ii) State Weierstrass's Approximation Theorem.

(iii) Define Radius of Convergence of a power series.

(iv) Find the radius of convergence of p-series $\sum (6+8i)z^n$.

(v) Find argument of i (x + iy) if argument (x + iy) = 2.

(vi) State Able's Theorem for uniform convegence of series.

(vii) Write f (z) = $x^2 - 3z + 2$ in the fork f (z) = u (x, y) + iv (x, y).

(viii) Define inverse of a bilinear transformation. 1×8=8