VECTOR CALCULUS - IV

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B carrying 8 marks each, and the entire Section C consisting of 8 short answer type questions carrying 1 mark each.

Section - A

1. (a) A town has 1,00,000 population out of which 55,000 read 'The Tribune' and 65,000 read 'The Hindustan Time' while 25,000 read both the newspapers. How many read neither The Tribune nor The Hindustan Times ?

   (b) Let $A = \{1, 2, 3\}, B_1 = \{1, 2\}, B_2 = \{2, 3\}$.
   (i) Find minsets and maxsets generated by B_1 and B_2.
   (ii) Write B_2 as minsets normal form. What is maxset normal form of B_2 ?

2. (a) Prove that Cartesian product of the countable sets is a countable set.
   (b) What is a Compact set ? Show that the set IR is not compact.

3. (a) Prove that the closure of A is the intersection of all closed supersets of A in R.
   (b) Prove that if a function is continous in the closed interval [a, b], then it is uniformly continous in [a, b].

4. (a) State and prove Heine Borel theorem.

   (b) Show that the set $S = \left\{\frac{3-x}{1-x},\ x > 0,\ x \neq 1\right\}$ is neither bounded above nor bounded below.

Section - B

5. (a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = t^2 + 5$, where t is the time. Find the components of its velocity and acceleration at time t = 1 in the direction $\hat{i} + \hat{j} + 3\hat{k}$.

   (b) Evaluate $\int_1^2 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right) dt$ where $\vec{r} = 2t^2\hat{i} + t\hat{j} - 3t^3\hat{k}$.

6. (a) Find the direction derivative of the function $\phi = x^2 + y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). Also find maximum value of the directional derivative P (1, 2, 3).

   (b) Show that the vector field represented by $\vec{F} = (z^2 + 2x + 3y)\hat{i} + (3x + 2y + z)\hat{j} + (y + 2zx)\hat{k}$ is irrational but solenoidal.

7. (a) Verify Green's theorem in the plane for $\oint_C \{(x^2 - xy^3)dx + (y^2 - 2xy)dy\}$ where C is the square with vertices (0, 0); (2, 0); (2, 2); (0, 2).

(b) If $\vec{F} = (2x^2 + y^2)\hat{i} + (3y - 4x)\hat{j}$, evaluate $\int \vec{F}.d\vec{r}$ around the triangle ABC whose vertices are A (0, 0); B (2, 0) and C (2, 1).

8. (a) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le x \le c$.

Section - C

9. Attempt all the following :

(a) Write the set $\{x : x$ is a +ve integer and $x^2 < 30\}$ in the roaster form.

(b) With the help of Venn diagram, show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(c) What is a Countable set ? Give an example.

(d) Find the l.u.b. and g.l.b. in $\left\{\frac{2x+1}{x+5} : |x - 4| < 2\right\}$.

(e) Solve that $dV - = d\vec{r}.\nabla V$.

(f) Find div. $\vec{F}$ where $\vec{F}$ = grad $(x^3 + y^3 + z^3 - 3xyz)$.

(g) State Green's theoprem in a plane.

(h) Prove that the whole line in $\mathbb{R}$ is an open set.