ALGEBRA

Paper-I
Semester-V

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt two questions each from Sections A and B carrying 5½ marks each and the entire Section C consisting of 10 short answer type questions carring 1¼ mark each.

SECTION-A

1. (a) Prove that set of positive integers which are less than n and co-prime to n forms an abelian group under operation of multiplication modulo n. 3
   (b) If a, be any two elements of a group G such that ab = ba and (0(a), 0(b)) = 1 then prove that 0(ab) = 0(a)0(b). 2½
2. (a) If H and K are finite subgroups of a group G, then $0(HK) = \frac{0(H)0(K)}{0(H0K)}$. 3
   (b) Show that the set of n-nth roots of unity froms an cyclic group under multiplication. 2½
3. (a) If H is a subgroup of G, then prove that G is equal to union of all right coset of H in G. 3
   (b) Let Q* denotes the set of all rational numbers except-1. Show that Q* forms an infinite abelian group under the operation * defined by a * b = a + b + ab, for a, b ∈ Q*. 2½
4. (a) Let G be a finite group and a ∈ G. Then 0(a)/0(G) i.e., the order of an element of a group is a divisor of the order of the group. 3
   (b) If N is a normal subgroup of a group G and H be any subgroup of G_n. V, then $(HN)/N \cong H/(H \cap N)$. 2½

SECTION-B

5. (a) If R is a ring in which $x^2 = x$ for all x ∈ R, prove that R is a commutative ring of characteristic 2. 3
   (b) If I and J are two ideals of a Ring R, then IJ is an ideal of R. Moreover $IJ \subseteq I \cap J$. 2½
6. (a) Let I = (a), J = (b) be two ideals of the ring Z of integers, where a, b are positive integers. Determine :
   (i) I + J
   (ii) IJ. 3
   (b) If A, B are any ideals of ring R and $B \subseteq A$, then A/B is an ideal of R/B. 2½
7. (a) Show that 12Z is an ideal of the ring 3Z. Describe the quotient ring $^{3Z}/_{12Z}$. 3
   (b) Show that relation of Isomorphism in the set of all rings is an equivalence relation. 2½
8. (a) If I and J be two ideals of ring R, then $I/(I \cap J) \cong I + J/J$. 3
   (b) Show that the mapping $f : Z[\sqrt{2}] \rightarrow M_2(R)$ defined by $f(a + b\sqrt{2}) = \begin{bmatrix} a & 2b \\ b & a \end{bmatrix}$ is a homomorphism of rings. Find Ker f. 2½

SECTION-C

10x1¼=14

9. Answer the following questions :
   (i) Prove that in a finite ꞏ ꞏ ꞏup order of every element exist.
   (ii) State and prove Reversal law of product.
   (iii) Every quotient of cyclic group of cyclic.
   (iv) By example show that union of two subgroups is not a subgroup.

(v) Prove that centre of a ring R is a subring of R.
(vi) Intersection of two left ideal of a ring is left ideal.
(vii) Define Associate and Irreducible elements of a Ring.
(viii) Find units of the Ring Z[i].
(ix) Define Division ring with example.
(x) Show that (Z, +) is a group.