Discrete Mathematics -IV
Semester - VI

Time Allowed : Three Hours **Maximum Marks : 40**

Note :- Attempt two questions each from Section A and B carrying 8 marks each, and the entire Section C consisting of 10 short answer type questions carrying 8 marks in all.

Section - A

1. Find the generating function and the sequence for the recurrence relation $S_n - 6s_{n-1} + 8s_{n-2} = 0$; for $n \geq 2$ and $s_0 = 10$, $s_1 = 25$.
2. Find the value of constants a, b and c, if the numeric function of recurrence relation. $as_n + bs_{n-1} + cs_{n-2} = 6$ is $3^n + 4^n + 2$
3. If $S_n = 2^n$, $T_n = 3^n$, then find the convolution S*T and verify that G(S*T, z) = G(S, z). G(T, z)
4. (a) Let $(a_0, a_1, \ldots\ldots\ldots, a_n, \ldots..)$ be an arbitrary numeric function and $(1, 1, \ldots\ldots\ldots, 1, \ldots.)$ be numeric function. Find the generating function if c is the convolution of these two numeric functions.

   (b) Let a and b denote the positive integers. Suppose a function Q is defined as $Q(a,b) = \begin{cases} 0 & ; \text{if } a \leq b \\ (a-b,b) & ; \text{if } b \leq a \end{cases}$ find Q(2,3) and Q(14, 3)

Section-B

5. Let B be a finite Boolean algebra and let A be the set of atoms of B. If P(A) is the Boolean algebra of all subsets of the set A of atoms, then show that the mapping $f : B \rightarrow P(A)$ is an isomorphism.
6. (a) Show that $(\mathbb{N}, \text{gcd}, \text{lcm})$ is a distributive lattice where $\mathbb{N}$ is a set of natural numbers.

   (b) State and prove De-Morgan's Laws in a Boolean algebra.
7. (a) Define Group. Show that $\mathbb{Z}$, the set of integers, is a group under binary operation (*) defined as $a * b = a + b - 1$

   (b) Draw the switiching circuit for $f = a + a'(a + b) + ab$. Simplify it and draw equivalent circuit.
8. Simplify the following Boolean function and realize the logic diagram of the reduced function with the help of NAND gate only.

   $F(A,B,C,D) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + ABC\bar{D} + ABCD$

Section-C

9. Do as directed :

   (a) Give an example of semi-group which is not a group.

(b) Define Isomorphic Boolean algebra with a suitable example.

(c) Show that idempotent laws follow from absorption laws in lattices.

(d) Write generating function of sequence $2^n\left[3+2(-1)^n\right]$.

(e) Determine C(5, 2) by the recursive definition of binomial coefficient.

(f) Check whether the set L = {1, 2, 3, 5, 30} under divisibility relation is a distributive lattice.

(g) Check whether D_{20} is a Boolean algebra or not.

(h) Define Boolean function with an example.

(i) Find the dual of Boolean equation $(a*1)*(0+a')=0$

(j) Determine all the Boolean sub-algebra of Boolean algebra D_{30}.