DISCRETE MATHEMATICS - II

Paper-IV : Semester-VI

Time Allowed : Three Hours **Maximum Marks : 40**

Note : Attempt five questions in all. Select two questions each from Section A and B and Q. No. IX of Section-C. All questions carry equal marks.

SECTION-A

I. The solution of recurrence relation

$$C_0a_r + C_1a_{r-1} + C_2a_{r-2} = f(r)$$

is $3^r + r^r + 2$.

Given that $f(r) = 6 \ \forall$ a. Find C_0, C_1, C_2.

II. If $S(k) - 6S(k-1) + 5S(k-2) = 0$, $S(0) = 1$, $S(1) = 2$, what is the generating function of S, G (S, z) ? Find the sequence which satifies it.

III. Find a particular solution of

$$S(n) + 5S(n-1) + 6S(n-2) = 3n^2 - 2n + 1.$$

IV. (a) Derive a second order linear relation for $C(k) = 3^{k-1} + 2^{k+1} + k, k \geq 0$.

(b) Explain algorithm of solving nth order linear homogeneous recurrence relations.

SECTION–B

V. (a) Show a lattice L is non-distributive iff it contains a sub-lattice that is isomorphic to pentagon lattice.

(b) Let $[B; -, \vee \wedge]$ be any Boolean algebra of order 2. Let $f : B^k \rightarrow B$. Find the 16 possible functions.

VI. (a) If $D_6 = \{1, 2, 3, 6\}$ be a lattice under divisibility, then find the complement of each element of D_6 in $[D_6, \vee \wedge]$.

(b) Define Switching circuits. Give suitable example.

VII. (a) Prove by using Boolean algebra B that

(i) $a + bc = (a + b). (a + c)$. (ii) $a + a.c = a + c$.

(b) Explain design and implementation of Digital networks with example.

VIII. (a) Find the minterms of the Boolean expression $f(x_1, x_2) = x_1 \vee x_2$.

(b) Prove that the complement of every element in a Boolean algebra B is unique.

SECTION–C

IX. Attempt all the following :

(a) Find the sequence L by $L_0 = 5$ and for $k \geq 1$, $L_k = 2L_{k-1} - 7$, find L_4.

(b) Define Homogeneous Recurrence relation with example.

(c) By the recursive definition of binomial coefficient, find C(5, 3).

(d) Solve the recurrence relation $\sqrt{a_n} = \sqrt{a_{n+1}} + \sqrt{a_{n+2}}$.

(e) Write a short note on Analysis of algorithms-time complexity.

(f) Find the atoms of the following Boolean lattice :
{1, 2, 5, 10, 11, 22, 55, 110}.

(g) Define Duality and Complemented lattices.

(h) Find list all atoms of B_2^4.

(i) Show that if n is a positive integer and p^2/n, where p is a prime number, then D_n is not a Boolean algebra.

(j) Define Sub-Boolean Algebra.