1E2002

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B.Tech. I Sem. (Main/Back) Examination - 2014
102 Engineering Mathematics-I
Common to all Branches

Time : 3 Hours] [Total Marks : 80
[Min. Passing Marks : 24

Instructions to Candidates :
Attempt any five questions, selecting one question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

UNIT - I

1. (a) Find the asymptotes of the following curve:

$$x^3 - x^2y - xy^2 + y^3 + x^2 - y^2 - 1 = 0$$ **(8)**

(b) The tangents at two points A and B on the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ are at right angles. If ρ_1 and ρ_2 be the radii of curvature at these points, then prove that: $\rho_1^2 + \rho_2^2 = 16a^2$. **(8)**

OR

1. (a) Find the points of inflexion on the curve:
$y^2 = x(x+1)^2$ **(8)**

(b) Trace the curves $r^2 = a^2\cos 2\theta$. **(8)**

UNIT - II

2. (a) If $z(x+y) = x^2 + y^2$, show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$ **(8)**

(b) If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$$ **(8)**

OR

(a) Find the points where the function
$x^3 y^2 (1 - x - y)$
has maximum or minimum value and also find the value of the function at these points. **(8)**

(b) Find the minimum/extreme value of $x^2 + y^2 + z^2$, subject to the condition $ax + by + cz = p$. **(8)**

UNIT - III

(a) Find the volume of the solid generated by the revolution of the curve
$y(a^2 + x^2) = a^3$ about its asymptotes. **(8)**

(b) Evaluate: $\iint_A y\,dx\,dy$
where A is the region of integration bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. **(8)**

OR

3. (a) Change the order of integration in the following double integral:

$$\int_0^{a\cos\alpha} \int_{x\tan\alpha}^{\sqrt{a^2-x^2}} f(x,y)dxdy$$

(b) Prove that : $B(m,n) = \frac{\lceil m \lceil n}{\lceil (m+n)}, m>0, n>0$.

UNIT - IV

4. Solve the following differential equations:

(a) $\left(x+2y^3\right)\frac{dy}{dx} = y$

(b) $2\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$

(c) $\left(xy^2+2x^2y^3\right)dx + \left(x^2y - x^3y^2\right)dy = 0$

OR

4. Solve the following differential equations:

(a) $\left(D^3-2D^2+4D-8\right)y = 0$

(b) $\left(D^2-4D+4\right)y = e^{2x} + \sin 2x$

(c) $\left(D^3-D^2-6D\right)y = 1+x^2$

UNIT - V

5. (a) Solve: $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = x^5$

(b) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2x^2$

OR

5. (a) Solve the differential equation:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$$

by using the method of variation of parameters.

(b) Solve

$$x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$$