2E2002

B.Tech. II Semester (Main/Back) Examination, June/July - 2016

202 Engg. Mathematics - II

Time : 3 Hours

Maximum Marks : 80

Min. Passing Marks : 26

Instructions to Candidates:

*Attempt any **five** questions, selecting one question from **each unit**. All questions carry **equal** marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)*

Unit - I

1. a) A plane passes through a fixed point (a,b,c) and cut the axis in A,B,C. Show that the locus of the centre of the sphere OABC is **(8)**

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$$

b) Two spheres of radii r_1 and r_2 cut orthogonally, prove the radius of their common circle is (8)

$$\frac{r_1 r_2}{\sqrt{r_1^2+r_2^2}}$$

OR

1. a) Define right circular cone. Find the equation of the right circular cone whose vertex is origin, axis is x - axis and semi vertical angle is α. (2+6=8)

b) Define right circular cylinder. Find the equation of a right circular cylinder whose axis is

$$\frac{x-2}{2}=\frac{y-1}{1}=\frac{z}{3}$$

and which passes through (0,0,1) (2+6=8)

Unit - II

Find the rank of the following matrix by reducing it to the normal form :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$ **(8)**

b) Test the consistency of the following system of equations and if possible solve it:

$$2x-3y+7z=5$$
$$3x+y-3z=13$$
$$3x+19y-47z=32$$ **(8)**

OR

2\. a) Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$$ **(8)**

b) State cayley Hamilton Theorem, verify it for the matrix.

$A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and find A^{-1}. **(2+6=8)**

Unit - III

3\. a) A particle moves on the curve $x=2t^2, y=t^2-4t \quad z=3t-5$, where t denote time. Find the components of velocity and acceleration at t = 1 in the direction of vector $\hat{i}-3\hat{j}+2\hat{k}$. **(8)**

b) Prove that :

i) $\nabla^2(r^n)=n(n+2)r^{n-2}$, if $r=|\bar{r}|=\sqrt{x^2+y^2+z^2}$ and $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$.

ii) $Curl(\bar{a}\times\bar{r})=2\bar{a}$, if $\bar{a}$ is a constant vector. **(4+4=8)**

OR

3. a) If $\bar{a}$ and $\bar{b}$ are differentiable vector functions, then show that :

i) $div(\bar{a}\times\bar{b}) = \bar{b}.curl\,\bar{a} - \bar{a}.\,curl\;\bar{b}$

ii) $div\,curl\,\bar{a} = 0$ **(5+3=8)**

b) Evaluate $\int_c \bar{F}.d\bar{r}$, where $\bar{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$, curve c is rectangle in the xy - plane bounded by x = 0; x = a; y = 0; y = b. **(8)**

Unit - IV

4. a) Evaluate $\iint_s \bar{F}.\hat{n}\,ds$ by using Gauss's divergence theorem for $\bar{F} = xy\hat{i} + z^2\hat{j} + 2yz\hat{k}$ on the tetrahedron x = y = z = 0, x + y + z = 1. **(8)**

b) State stoke's theorem. Verify Green's theorem in the plane for $\oint_C [(xy+y^2)dx + x^2dy]$, where c is the closed curve of the region bounded by $y = x^2$ and $y = x$. **(2+6=8)**

OR

4. a) Obtain the Fourier series for the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and deduce the following :

i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots = \frac{\pi^2}{6}$

ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots\dots\dots = \frac{\pi^2}{12}$ **(5+3=8)**

b) Express f(x) in a fourier series upto the second harmonic for the following data :

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x) :	1.98	2.15	2.77	-0.22	-0.31	1.43	1.98

(8

Unit - V

5\. a) Solve the following differential equation in series. **(8)**

$$\frac{d^2y}{dx^2}+x^2y=0$$

b) Solve : **(8)**

$$x(y^2-z^2)p-y(z^2+x^2)q=z(x^2+y^2)$$

OR

5\. a) Solve : **(8)**

$$x^2p^2+y^2q^2=z^2$$

b) Find a complete integral of

$$q=(z+px)^2$$

by using charpit's method. **(8)**