

3E1626

B.Tech. (Sem.III) (Main/Back) Examination-2014 Civil Engineering 3CE6 Advanced Engineering Mathematics

Time : 3 Hours

Total Marks : 80
Min. Passing Marks : 24

Instructions to Candidates :

Attempt any five questions selecting one question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

UNIT - I

1. (a) Find the Fourier series to represent $f(x) = x - x^2$ in the interval $-1 < x < 1$. (8)
- (b) Obtain the first three cosine terms and the constant term in the Fourier series for y , where
- | | | | | | | |
|------|---|---|----|---|---|---|
| $x:$ | 0 | 1 | 2 | 3 | 4 | 5 |
| $y:$ | 4 | 8 | 15 | 7 | 6 | 2 |
- (8)

OR

1. (a) Use convolution theorem to evaluate $Z\left[\frac{z^2}{(z-a)(z-b)}\right]$ (8)
- (b) Obtain $Z[\sin n\theta]$, $Z[\cos n\theta]$ and hence find $Z[a^n \sin n\theta]$ and $Z[a^n \cos n\theta]$. (8)

UNIT - II

2. (a) Find Inverse Laplace transform of $\frac{s}{s^4 + 4a^4}$ (8)
- (b) Use L.T. theory to solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = t$; given $y(0) = -3, y(1) = 1$ (8)

OR

2. (a) If $L[f(t)] = \bar{f}(s)$, then prove that $L[tf(t)] = -\frac{d}{ds}\bar{f}(s)$. Hence, find the Laplace transform of $t^2 \cos at$. (8)
- (b) Find the bounded solution $y(x, t), 0 < x < 1, t > 0$ of the boundary value problem.
- $$\frac{\partial y}{\partial x} - \frac{\partial y}{\partial t} = 1 - e^{-x}, y(x, 0) = x$$
- (8)

UNIT - III

1. (a) Find $f(x)$ if its Fourier sine transform is $\frac{1}{s} e^{-s}$. Hence deduce $\bar{F}_s^{-1}\left[\frac{1}{s}\right]$. (8)
- (b) Use Fourier transform theory to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given that $u_x(0, t) = 0$ and $u(x, 0) = \begin{cases} x, 0 \leq x \leq 1 \\ 0, x > 1 \end{cases}$, $u(x, t)$ is bounded and $x > 0, t > 0$. (8)

OR

1. (a) Find the Fourier transform of:
- $$f(x) = \begin{cases} 1 - x^2, |x| < 1 \\ 0, |x| > 1 \end{cases}$$
- (8)

(b) Use Fourier transform theory to solve :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, t > 0 \text{ given that}$$

$$u(x, 0) = f(x) = \begin{cases} u_0, & |x| < a \\ 0, & |x| > a \end{cases}$$

UNIT - IV

4. (a) Use Lagrange's formula to find $f(x)$ from the following data.

x:	0	1	4	5
f(x):	4	3	24	39

(b) A slider in a machine moves along a fixed straight rod. Its distance x (cm) along the rod is given below for values of time t (secs.).

t:	0	0.1	0.2	0.3	0.4	0.5	0.6
x:	30.28	31.43	32.98	33.54	33.97	33.48	32.13

Evaluate : (i) $\frac{dx}{dt}$ for $t = 0.1$ and $t = 0.3$.

(ii) $\frac{dx}{dt}$ for $t = 0.5$ and also $\frac{d^2x}{dt^2}$ for all above points

OR

4. (a) Compute u_{12} from the following data

x:	10	11	12	13	14
$10^5 u_x$:	23967	28060	31788	35209	38368

State the formula used, why?

(b) Evaluate Numerically $\int_0^1 \frac{dx}{1+x^2}$ using :

(i) Simpson's $\frac{1}{3}$ rule and

(ii) Simpson's $\frac{3}{8}$ rule. Find approximate value of π .

UNIT - V

5. (a) Employ Picard's method to obtain correct to four places of decimals solution to the differential equation: $\frac{dy}{dx} = y - x$ with $y = 0$ when $x = 0$ for $x = 0.4$

(b) Solve by the modified Euler's method the differential equation : $\frac{dy}{dx} = y^2 - \frac{y}{x}$, given that, $y = 1$ when $x = 1$ determine y for $x = 1.1, (0.1), 1.4$.

OR

5. (a) If $\frac{dy}{dx} = x + y^2$, use Runge-Kutta method find an approximate value of y for $x = 0.2$, given that $y = 1$ when $x = 0$. ($h = 0.1$).

(b) Use Milne's Predictor-Corrector method to obtain $y(0.4)$ for the following differential equation $\frac{dy}{dx} = 2e^x - y$ given that

x:	0	0.1	0.2	0.3
y:	2	2.01	2.04	2.09