4E4161

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B.Tech. IVSemester(Main/Back) Examination, June/July - 2015

Computer Science and Engineering

4CS2A Discrete Mathematical Structures

Common with IT

Time : 3 Hours **Maximum Marks : 80**

Min. Passing Marks : 26

Instructions to Candidates:

*Attempt any **five** questions, selecting **one** question from **each unit**. All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

Unit - I

1. a) State and prove the Principle of Inclusion and Exclusion for three sets A,B and C. **(4)**

b) There are 250 students in a computer Institute of these 180 have taken a course in Pascal, 150 have taken a course in C++, 120 have taken a course in Java. Further 80 have taken Pascal and C++, 60 have taken C++ and Java, 40 have taken Pascal and Java and 35 have taken all 3 courses. So find-

i) How many students have not taken any course?

ii) How many study at least one of the languages?

iii) How many students study only Java?

iv) How many students study Pascal and C++ but not Java? **(8)**

c) Let $A=\{1,1,1,2,2,3,4,4\}$ and $B=\{1,2,4,4,5,5,5\}$. Find $A\cup B, A\cap B, A-B$ and $A+B$. **(4)**

OR

1. a) Let f: $R\rightarrow R$ be a function defined as f(x)=3x+5 and g: $R\rightarrow R$ be another function defined as g(x)=x+4. Find $(gof)^{-1}$ and $f^{-1}og^{-1}$ and verify $(gof)^{-1}=f^{-1}og^{-1}$ **(6)**

b) Any 7 numbers are choosen from 1-12. Show that,

i) Two of them will add to 13.

ii) There are two relative prime integers. **(4)**

c) Define the followings with example:

i) Floor function

ii) Ceilling function

iii) Remainder function. **(6)**

Unit - II

2. a) Let $A = \{1,2,3,4\}$ *and* $R = \{(1,3),(3,2)(2,4)(3,1)(4,1)\}$. Find the transitive closure of R using Warshall's algorithm. **(8)**

b) Define the followings with example:

i) Equivalence relation

ii) Partial order relation

iii) Total order relation

iv) Cross partition of a set. **(8)**

OR

2. a) Let R be a relation defined on a set of ordered pairs of positive integers such that for all $(x,y),(u,v) \in Z^+ \times Z^+$, (x,y) R (u,v) if and only if $\frac{u}{x} = \frac{v}{y}$. Determine whether R is an equivalence relation. **(8)**

b) Let $A = \{1,2,3,4\}$ *and* $R = \{(a,b) : a+b > 4\}$ be a relation on A. Draw the graph of the relation R. **(4)**

c) Let R be an equivalence relation on a set of positive integers defined by x R y if and only if $x \equiv y (\text{mod } 3)$. Then, find the equivalence class of 2 and also find the partition generated by the equivalence relation. **(4)**

Unit - III

3. a) *Let* $a_n = a_{n-1} + a_{n-2}$ *for* $n \geq 3$ with the initial conditions $a_1 = a_2 = 1$, then prove that $2^{n-1} a_n \equiv n (\text{mod } 5), \forall n \geq 1$ **(4)**

b) State and prove Euclidean Algorithm for integers. **(8)**

c) Use binary search algorithm to search the list X={3,5,8,13,21,34,55,89} for key=5 **(4)**

OR

3. a) Prove that $7^{2n}+2^{3n-3}.3^{n-1}$ is divisible by 25 for all positive integers. **(4)**

b) State and prove Division Algorithm for integers. **(8)**

c) Use bubble sort to put 3,2,4,1,5 into searching order. **(4)**

Unit - IV

4. a) Define the followings with example:

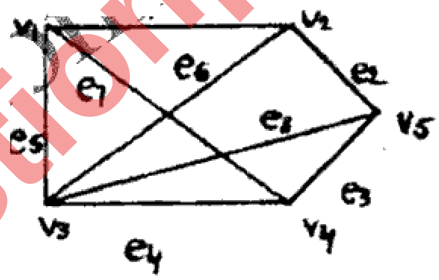
i) Complete graph

ii) Bipartite graph

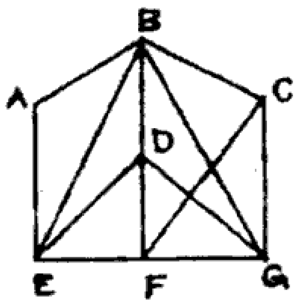
iii) Complete Bipartite graph

iv) Weighted graph **(8)**

b) Define spanning tree in a graph. Find five spanning trees for the graph shown in figure and write the sets of branches and chords corresponding to these spanning trees. **(8)**



4. a) Apply a breadth-search algorithm to explore all the vertices from the vertex A of the graph given in figure and find the breadth-first search tree. **(8)**



b) Prove that a disconnected simple graph G with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. **(8)**

Unit - V

5\. a) Explain the following for propositions with example:-

i) Logical Equivalence **(2)**

ii) Tautological Implication **(2)**

iii) Normal Forms **(4)**

b) Check the validity of the following argument:

If I go to school, then I attend all classes. If I attend all classes, then I get A grade. I do not get grade A and I do not feel happy. Therefore, if i do not go to school then, I do not feel happy. **(4)**

c) Find the DNF of following:

i) $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim P \vee \sim P))$

ii) $\sim(P \rightarrow (Q \wedge R))$. **(4)**

OR

5\. a) Determine whether the conclusion C follows logically from the premises H_1, H_2 and H_3.

$$\begin{array}{l} H_1 : P \vee Q \\ H_2 : P \rightarrow R \\ \underline{H_3 : \sim Q \vee S} \\ C : S \vee R \end{array}$$ **(4)**

b) Explain the followings:

i) Argument **(2)**

ii) Predicates **(2)**

iii) Quantifiers **(4)**

c) Without constructing the truth table, show that $(\sim P \wedge (P \vee Q)) \rightarrow Q$ is a tautology. **(4)**
