3E1616

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B.Tech. III Semester (Main&Back) Examination - 2016
Applied Elect. & Inst. Engg.
3AI1 Mathematics III/Advanced Engg. Mathematics - I
EC, EIC, BM, AI, CR, PE, PC

Time : 3 Hours

Maximum Marks : 80
Min. Passing Marks : 26

Instructions to Candidates:

*Attempt any **five** questions, selecting **one** question from **each unit**. All questions carry **equal** marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)*

Unit - I

1. a) Find the Laplace transform of the following functions :

   i) $\cos(at)\cosh(at)$

   ii) $t^2 e^t \sin 3t$

   b) Use Laplace transform theory to solve the equation

   $$\left(D^2-3D+2\right)x=1-e^{2t},\ x(0)=1,\ x'(0)=0.\ \text{Where } D=\frac{d}{dt}.$$

OR

1. a) Find the inverse Laplace transform of the following functions :

   i) $\dfrac{4s+5}{(s-1)^2(s+2)}$

   ii) $\dfrac{e^{-2s}}{s-3}$

   b) Use Laplace transform theory to solve

   $$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\ u(x,0)=3\sin 2\pi x,\ u(0,t)=0,\ u(1,t)=0,\ \text{where } 0<x<1,\ t>0.$$

1. a) Find the Fourier series for $f(x) = x + x^2, -\pi < x < \pi$. Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + ...$

b) i) Find z-transform of $\{a^k\}; k \geq 0$

ii) Find Z-transform of $\{f(k)\}$ where $f(k) = \begin{cases} 4^k; k < 0 \\ 3^k; k \geq 0 \end{cases}$

OR

2. a) The following table gives the variations of a periodic current over a period

t(secs)	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A (amps)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show by harmonic analysis that there is a direct current part of 0.75 amp. in the variable current and obtain the amplitude of the first harmonic.

b) Find the sequence $\{f(k)\}$ if $f(z) = \frac{z}{z-a}$ for

i) $|z| > |a|$

ii) $|z| < |a|$

Unit - III

3. a) Find the Fourier sine and cosine transform of $f(x)$, where

$f(x) = 1, \; for \; 0 < x < a$
$= 0, \; for \; x > a$

b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given that $u_x(0,t) = 0$ and $u(x,0) = \begin{cases} x, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$ u (x,t) is bounded and x > 0, t > 0.

OR

3. a) Find the Fourier sine and cosine transform of $f(x) = e^{-x}, x \geq 0$. Also, show that $\int_0^\infty \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}, m > 0$.

b) Solve the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, t > 0$. Subject to $u(x,0) = f(x)$,

where $f(x) = \begin{cases} u_0, & |x| < a \\ 0, & |x| > a \end{cases}$

Unit - IV

4. a) Define an analytic function. If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = e^x(\cos y - \sin y)$, find f(z) in terms of z.

b) Evaluate the following integral by using Cauchy's integral formula

$\int_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|Z| = 3$.

OR

4. a) Show that the function $u + iv = f(z)$, where

$$f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2} & , z \neq 0 \\ 0 & , z = 0 \end{cases}$$

Satisfies the Cauchy - Riemann equations at the origin, yet $f'(0)$ does not exist.

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$.

Unit - V

5. a) Expand $\frac{1}{z(z^2 - 3z + 2)}$ in Laurent series for the region :

i) $0 < |z| < 1$

ii) $1 < |z| < 2$

iii) $|z| < 2$

b) Determine the poles of the function $f(z) = \frac{z^2}{(z+2)(z-1)^3}$ and the residue at each pole. Hence evaluate $\int_c f(z) dz$, where c is the circle $|z| = 1.5$

OR

a) Expand cosz about the point $z = \frac{\pi}{2}$ by Taylor's theorem.

b) Evaluate the following integral by contour integration $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$