4E4131

Roll No

Total No of Pages: 4

4E4131

B.Tech. IV-Sem (Main & Back) Exam; June-July 2016
Electronics & Communication
4EC2A Random Variables & Stochastic Processes

Time: 3 Hours

Maximum Marks: 80
Min. Passing Marks (Main & Back): 26
Min. Passing Marks (Old Back): 24

Instructions to Candidates:-

*Attempt any **five questions**, selecting **one question** from **each unit**. All Questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*
Units of quantities used/ calculated must be stated clearly.

Use of following supporting material is permitted during examination.
(Mentioned in form No.205)

1. NIL 2. NIL

UNIT-I

Q.1 (a) Prove that $2^n - (n+1)$ equations are needed to establish the mutual independence of n – events. [8]

(b) The age of a person when he dies is denoted by t. The probability that $t \le t_0$ is given by the following equation

$$P(t \le t_0) = \int_0^{t_0} A(t)\,dt$$

Where A(t) is a function determined from mortality records. The curve between A(t) and t is given in Fig.1(b) for $0 \le t \le 100$ years and A(t) is given as $A(t) = 3 \times 10^{-9}\, t^2 (100 - t)^2$; $0 \le t \le 100$years.

Determine the probability that a person will die between the ages of 60 & 70 assuming that he was alive at 60. [8]

Fig .1(b)

OR

Q.1 (a) In a system, there are n components connected in series. This system works successfully when all units (components) work successfully. The operation of each component is independent to each other. The probability of successful operation of the components is p_i where $i = 1, 2, 3, \ldots\ldots, n$. Find the probability that the system functions satisfactorily. [8]

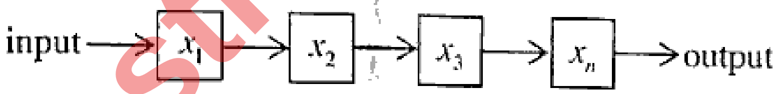

fig.1(a)

(b) State & explain the theorem of total probability & Bayes Theorem. [8]

UNIT-II

Q.2 (a) Explain all the properties of conditional Distribution. [6]

(b) Determine the mean and variance of the random variable X of the following distribution

(i) Uniform distribution [5]

(ii) Exponential distribution [5]

OR

Q.2 (a) Determine the mean and variance of the random variable X of the following distribution

(i) Normal distribution [5]

(ii) Rayleigh distribution [5]

(b) Prove the reproductive property of independent Poisson Random Variable. Hence find the probability of 5 or more telephone calls arriving in a 9min. period in a collage switch board, if the telephone calls that are arrived at the rate of 2 every 3min. Follow a Poisson distribution. [6]

UNIT-III

Q.3 (a) Consider Z = X + Y, show that if X and Y are independent Poisson's RV's with parameters λ_1 and λ_2, respectively, then Z is also a Poisson Random Variable. [8]

(b) Let X and Y be the independent random variables with common parameters λ. Define U = X + Y, V = X – Y. Find the joint and marginal pdf of U and V. [8]

OR

Q.3 (a) A voltage V is a function of time t and is given by

$V(t) = X \cos wt + Y \sin wt$

In which w is a constant angular frequency and $X = Y = N(0, \sigma^2)$ and they are independent. [10]

(i) Show that V(t) may be written as

$V(t) = R \cos(wt - \theta)$

(ii) Find the pdfs of RV's R and θ and show that R and θ are independent.

(b) Define a two dimensional random variable. Give an example of the out – come of a random experiment, that is a two dimensional random variables. [6]

UNIT-IV

Q.4 (a) Consider a continuous random variable X, prove that

$$E[X] = \int_0^{\infty}[1 - F_X(x)]dx - \int_{-\infty}^{0} F_X(x)dx$$

Where $F_X(x)$ is the cdf of X. [6]

(b) Explain the followings;

(i) Liapounoff's form of CLT. [5]

(ii) Lindberg – Levy's form of CLT. [5]

Where CLT = Central Limit Theorem.

OR

Q.4 (a) Consider the random variable X whose characteristics function is given by

$$\varphi_X(w) = \begin{cases} 1 - |w| ; & |w| < 1 \\ 0 ; & |w| > 1 \end{cases}$$

Determine the pdf of X. [8]

(b) The Moment generating function of a random variable X is given by

$$M_X(\Omega) = \frac{5}{5 - \Omega}$$ [4]

Determine the standard deviation of X. [4]

(c) Write down all the properties of characteristics function $\phi_x(w)$. [8]

UNIT-V

Q.5 (a) Write and explain all the properties of power spectral density.

(b) Let X(t) be the WSS process with the auto correlation function given by

$$R_{XX}(\tau) = \left(\frac{A_0^2}{2}\right)\cos(w_0\tau)$$ [8]

Where A_o and w_o are constants. Determine the psd of X(t).

OR

Q.5 (a) In the figure given below, X(t) be a input voltage to a circuit and Y(t) be the output voltage. The process X(t) is a stationary random process with zero mean and auto correlation

$$R_{XX}(\tau) = e^{-\alpha|\tau|}$$ [10]

Determine E[Y(t)], $S_{YY}(w)$ and $R_{YY}(\tau)$.

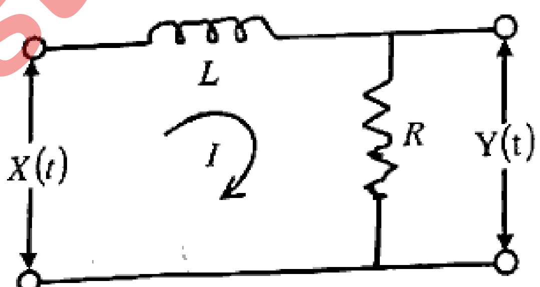

Fig.5(a)

(b) The psd of white noise $\left(\frac{N_0}{2}\right)$ is 6×10^{-6} W/ Hz., is applied to an ideal Low Pass Filter with power transfer function 1 and bandwidth W rad/sec. Find W so that output average noise power is 15 watt. [6]