3E1646

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B. Tech III Sem. (Main/Back) Exam. Jan. 2016
Electrical & Electronics Engineering
3EX6A Advanced Engineering Mathematics - I
Common to EE, EX

Time: 3 Hours

Maximum Marks: 80
Min. Passing Marks: 24

Instructions to Candidates:

*Attempt any **five questions**, selecting **one question** from **each unit**. All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*

Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL ______________ 2. NIL ______________

UNIT-I

Q.1 (a) Find Laplace transform of $t^2 H(t-3)$. [4]

(b) Find inverse Laplace transform of- $\tan^{-1}\frac{2}{s^2}$. [4]

(c) Solve the following differential equation using Laplace transform technique:

$(D^4 - 1)\,y = 1,\quad y(0) = y'(0) = y''(0) = y'''(0) = 0$ [8]

OR

Q.1 (a) Find Laplace transform of- $\frac{e^{-t}\sin t}{t}$. [4]

(b) Find inverse Laplace transform of $\frac{se^{-2s}}{s^2-1}$. [4]

(c) Solve the following partial differential equation using Laplace transform technique: [8]

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

subject to the constraints

$u(0, t) = u(5, t) = 0$; $u(x, 0)$ 10 sm $4\pi x$.

UNIT-II

Q.2 (a) Express the function –

$$f(x) = \begin{cases} \frac{2}{\pi}\sin x & ; 0 \le x \le \pi \\ 0 \quad ; & x > \pi \end{cases}$$

in integral form with the help of Fourier sine transform and hence evaluate

$$\int_0^\infty \frac{\sin(\pi\lambda)\sin(x\lambda)}{(1-\lambda^2)} d\lambda$$ [6+2=8]

(b) Find discrete Fourier transform of the sequence $\{d_q\} = \{1, 2, 3, 4\}$. [8]

OR

Q.2 (a) Prove that - $\int_0^\infty \frac{\cos x\lambda}{(1+\lambda^2)} d\lambda = \frac{\pi e^{-x}}{2}$, $x \ge 0$ [8]

(b) Solve the following partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \; ; x > 0, t > 0$$

subject to the constraints

$u = 0$, when $x = 0, t > 0$

$$u = \begin{cases} 1 & ; \quad 0 < x < 1 \\ 0 & \quad x \ge 1 \end{cases}, \text{ when } t = 0$$ [8]

UNIT-III

Q.3 (a) Find Fourier series to represent the function – [8]

$$f(x)=\begin{cases}0 & ; -\pi<x<0\\ \frac{\pi x}{4} & ; 0<x<\pi\end{cases},$$

Hence deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots$

(b) Find the curve through two points (x_1, y_1) and (x_2, y_2) which when rotated about the x- axis, gives minimum surface area. [8]

OR

Q.3 (a) Find half range cosine series for the function $f(x) = x(\pi - x)$; $x \in (0, \pi)$ [8]

(b) Find the shape of the plane curve of fixed length l so that it encloses maximum area. [8]

UNIT-IV

Q.4 (a) Show that the function $f(z)=\sqrt{|xy|}$ is not regular at origin, although Cauchy Riemann equations are satisfied. [8]

(b) Find the image of the infinite step $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$. Also show the regions graphically. [6+2=8]

OR

Q.4 (a) Find the bilinear transformation that maps the points 0,- i, -1 in z- plane onto the points w = i, 1, 0. What are the invariant points of this transformation? [6+2=8]

(b) Using Cauchy's integral formula, evaluate –

$$\int\frac{z\,dz}{(z-2)\,(z-3)^2}$$ where C is the circle $|z-3|=\frac{1}{2}$. [8]

UNIT-V

Q.5 (a) Find Laurent's series for the function $f(z) = \frac{1}{z(1-z)}$ in the region -

(i) $|z+1| < 1$ [2½]

(ii) $1 < |z+1| < 2$ [2½]

(iii) $|z+1| > 2$ [3]

(b) Find the residue of f (z) at each of its singularity, where f (z) is -

(i) $\frac{1+e^z}{\sin z + z\cos z}$ [4]

(ii) $\frac{e^z}{z(z+1)^2}$ [4]

OR

Q.5 (a) Using Cauchy's residue theorem, evaluate the integral $\int_c \frac{z-3}{z^2+2z+5}\,dz$, where C is the curve $|z+1-i| = 2$. [8]

(b) Show that -

$$\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\,dx = \frac{5\pi}{12}$$ [8]
