(i) Printed Pages : 4]

(ii) Questions : 8]

Roll No. .....

Sub. Code : 0043

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B.A./B.Sc. (General) 1st Semester Examination

1127

MATHEMATICS

(Plane Geometry)

Paper : I

Time : 3 Hours] **[Max. Marks : 30**

Note :– Attempt *five* questions in all, selecting at least *two* questions from each Section.

Section–I

1. (a) Find the transformed equation of $17x^2 - 16xy + 17y^2 - 225 = 0$ when the axes are rotated through an angle of 45°.

(b) Show that if $ax^2 + 2hxy + by^2 = 1$ and $a'x^2 + 2h'xy + b'y^2 = 1$ represent the same conic and axes are rectangular, then $(a - b)^2 + 4h^2 = (a' - b')^2 + 4h'^2$. 3,3

2. (a) Prove that the angle between the lines joining the origin to the points of intersection of the straight line $y = 3x + 2$ with the curve $x^2 + 2xy + 3y^2 + 4x + 8y - 11 = 0$ is $\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)$.

(b) If p_1, p_2 be the lengths of perpendiculars drawn from point $(-1, 2)$ to the pair of lines $2x^2 - 5xy + 2y^2 + 3x - 3y + 1 = 0$, then prove that $p_1p_2 = \frac{12}{5}$. 3,3

3. (a) Find the equation of circle which passes through the point $(2, 0)$ and touches the straight line $x + 2y - 1 = 0$ at the point $(3, -1)$.

(b) The line $2x - y = 4$ meets the circle $x^2 + y^2 - 6x + 2y + 2 = 0$ at the points P and Q. If the tangents at P and Q meet at R. Find the coordinates of R. 3,3

4. (a) Find the equation of the circle which passes through the origin and cuts orthogonally each of the circles $x^2 + y^2 - 8x + 12 = 0$ and $x^2 + y^2 - 4x - 6y - 3 = 0$.

(b) Find the limiting points of the co-axial system determined by the circles $x^2 + y^2 - 6x - 6y + 4 = 0$, $x^2 + y^2 - 2x - 4y + 3 = 0$. 3,3

Section–II

5. (a) Prove that the locus of points such that two of the three normal from them to parabola $y^2 = 4ax$, coincide is $27ay^2 = 4(x - 2a)^3$.

(b) Find the locus of poles of normal chord of the parabola $y^2 = 4ax$. 3,3

6. (a) Prove that the locus of the foot of the perpendicular from the focus on any tangent to a parabola is the tangent at the vertex.

(b) If the normal at any point P of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the major axes in G, find the locus of mid-point of the chord PG. 3,3

7. (a) Find the lengths of the semi-diameter conjugate to the diameter $y = 3x$ of the ellipse $ax^2 + 4y^2 = 36$.

(b) Find the asymptotes to the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$. Also find the equation of its conjugate hyperbola. 3,3

8. (a) If e and e' be the eccentricities of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$ respectively, then prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$.

(b) Identify the curve $x^2 - 4xy + 4y^2 - 32x + 4y + 16 = 0$. Find its latus rectum, focus and directrix. 3,3