MATHEMATICS

Paper – I : (Advanced Calculus – I)

Time Allowed : 3 Hours **Maximum Marks : 30**

Note : Attempt five questions in all, selecting at least two from each Unit. All questions carry equal marks.

UNIT – I

1. (a) Show that : $\lim_{(x,y)\to(0,0)} \frac{x^4y^4}{(x^2+y^4)^3}$ does not exit.

(b) Discuss the continuity of the function at (0, 0)

$$f(x,y)=\begin{cases}\frac{xy^2}{x^2+y^2} & , (x,y)\neq(0,0)\\ 0 & , (x,y)=(0,0)\end{cases}$$ 3, 3

2. (a) If $z = xy\tan\frac{y}{x}$, prove that :

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2z$$

(b) If z is a function of x and y, prove that if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, then

$$\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$$ 3, 3

3. (a) Show that $f(x, y) = \sin x + \sin y$ is differentiable at every point of R^2.

(b) For $\phi = x^2y^3z^4$, find the directional derivative of ϕ at (2, 3, 1) in the direction making equal angles with x, y and z-axis. 3, 3

4. (a) Find constants a, b and c so that

$$\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$$

is irrotational.

(b) If $\vec{A}=xyz\,\hat{i}+xz^2\hat{j}-y^3\hat{k}$ and $\vec{B}=x^3\hat{i}-xyz\hat{j}+x^2z\hat{k}$, then find :

$$\frac{\partial^2\vec{A}}{\partial y^2}\times\frac{\partial^2\vec{B}}{\partial x^2}$$ at (1, 1, 0). 3, 3

UNIT–II

5. (a) If $H = f(x, y, z)$ is a homogeneous functions of x, y and z of degree n then : $x\frac{\partial H}{\partial x} + y\frac{\partial H}{\partial y} + z\frac{\partial H}{\partial z} = nH$

(b) Use Taylor's theorem to expand $xy^2 + 3x - 2$ in powers of $(x+2)$ and $(y-1)$. 3, 3

6. (a) If $x = r\cos\theta\cos\varphi$, $y = r\sin\theta\sqrt{1-m^2\sin^2\varphi}$ and $z = r\sin\phi\sqrt{1-n^2\sin^2\theta}$, where $m^2 + n^2 = 1$, then show that :

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = \frac{r^2(m^2\cos^2\varphi + n^2\cos^2\theta)}{\sqrt{(1-m^2\sin^2\varphi)(1-n^2\sin^2\theta)}}.$$

(b) Show that the functions $u = x^3 + x^2y + x^2z - z^2(x+y+z)$ $v = z + x$, $w = x^2 - z^2 + xy - zy$ are not independent of one another. Also find the relation between them. 3, 3

7. (a) Find the envelope of the family of lines $x\cos^3\theta + y\sin^3\theta = a$, θ being the parameter.

(b) Find the evolute of the parabola $y^2 = 4ax$ regarding it as an envelope of its normal. 3, 3

8. (a) Fine the maximum and minimum values of the function : $f(x, y) = \sin x + \sin y + \cos(x+y)$

(b) Let $f : R^3 \to R$ be defined by $f(x, y, z) = xyz$. Determine x, y, z for maximum of f subject to condition $xy + 2yz + 2zx = 108$. 3, 3