

MATHEMATICS Paper-II

(Differential Equations-I)

Time Allowed : 3 Hours

Max. Marks : 30

Note : Paper is divided into two Units : Unit-I and Unit-II. Each Unit contains four questions. Students are required to attempt five questions selecting at least two questions from each Unit.

UNIT-I

1. (a) Find the necessary and sufficient condition that the equation $Mdx + Ndy = 0$ may be exact where M and N are functions of x and y with

the condition that $M, N, \frac{\partial M}{\partial y}, \frac{\partial N}{\partial x}$ are continuous functions of

x, y .

- (b) Solve and test for singular solution :

$$p^3 - 4pxy + 8y^2 = 0 \text{ where } P = \frac{dy}{dx}.$$

3+3=6

2. (a) Solve the differential equation :

$$y'' + y = x e^x \sin 2x.$$

(b) Solve $(p-1)e^{4x} + p^2 e^{2y} = 0$ where $P = \frac{dy}{dx}$.

3+3=6

3. (a) Solve the differential equation : $\frac{d^2y}{dx^2} + a^2y = \sec ax.$

(b) Show that the system of confocal conics $\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1$ is self orthogonal.

3+3=6

4. (a) Solve : $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0.$

(b) Find the orthogonal trajectory of $r^n = a^n \cos n\theta.$

3+3=6

UNIT-II

5. (a) Solve : $(x^2D^2 + 3xD + 1)y = (1-x)^{-2}$ where $D = \frac{d}{dx}.$

(b) Solve : $\sqrt{x} \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} + 3y = x, x > 0.$

3+3=6

6. (a) Solve : $(D^2 - 1)y = 2(1 - e^{-2x})^{-1/2}$ by method of variation of parameters.

(b) Find the particular solution of the Linear system

$$\frac{dx}{dt} = -2x + 7y$$

$$\frac{dy}{dt} = 3x + 2y \text{ where } x(0) = 1 \text{ and } y(0) = -1.$$

3+3=6

7. (a) Using operator method, find the general solution of the linear system

$$\frac{dx}{dt} + \frac{dy}{dt} - x - 6y = e^{3t}, \quad \frac{dx}{dt} + 2\frac{dy}{dt} - 2x - 6y = t.$$

(b) Solve : $(3x+2)^2 \frac{d^2y}{dx^2} + 5(3x+2) \frac{dy}{dx} - 3y = x^2 + x = 1.$

3+3=6

8. (a) Solve: $\frac{d^2 y}{dx^2} - \cot x \frac{dy}{dx} - (1 - \cot x) y = e^x \sin x.$

(b) Solve: $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x.$

3+3=6