maximum ...

MATHEMATICS

Paper – II : Differential Equations – I

Time Allowed : 3 Hours

Maximum Marks : 30

Note : Attempt five questions in all, selecting at least two from each Unit. Each question carries 6 marks.

UNIT – I

1. (a) Define exact differential equation and solve $(\cos x \sinh y + 1)\, dx + \sin x \cosh y\, dy = 0$ by proving it is exact.   1+2

   (b) Solve : $x^2 y\, dx - (x^3 + y^3)\, dy = 0$.   3

2. (a) Solve : $y - 2px = \tan^{-1}(xp^2)$

(b) Solve : $(px - y)(py + x) = h^2 p$ 3, 3

3. (a) Find singular solution of : $3y = 2px - \dfrac{2p^2}{x}$

(b) Find orthogonal trajectories of $y^2 = 4ax$.   3, 3

4. (a) Solve : $(D^2 + 4)y + \cos 2x + e^x$

(b) Solve : $(D^2 - 4D + 4)y = \sinh 2x$ 3, 3

UNIT–II

5. (a) Solve the differential equation :
$(x^3 D^3 + 6x^2 D^2 + 4xD - 4)y = (\log x)^2$

(b) Solve : $\{(2x - 1)^3 D^3 + (2x - 1)D - 2\}y = 0$ 3, 3

6. (a) Use method of reduction of order to solve : $(D^2 + a^2)y = \sec ax$.   3

(b) Solve the method of variation of parameters : $(D^2 + 4)y = \sin 2x$.   3

7. (a) Verify that $y = e^x$ is solution of $(x - 1)y'' - xy' + y = 0$ and use this fact to find general solution of $(x - 1)y'' - xy' + y = 1$.

(b) Solve : $[(x^3 - 4x)D^3 + (9x^2 - 12)D^2 + 18xD + 6]y = 0$ 3, 3

8. (a) Solve : $\dfrac{dx}{dt} = ax + by$ and $\dfrac{dy}{dt} = bx + ay$

(b) Use operator method to find general solution of the linear system :

$$2\dfrac{dx}{dt} + \dfrac{dy}{dt} + x + y = t^2 + 4t$$

$$\dfrac{dx}{dt} + \dfrac{dy}{dt} + 2x + 2y = 2t^2 - 2t$$ 3, 3