MATHEMATICS Paper–III

(STATICS)

Time Allowed : 3 Hours **Max. Marks : 30**

Note : Attempt five questions selecting at least two questions from each Unit. Each question will carry 6 marks.

UNIT–I

1. (a) Prove that direction of the resultant of two concurrent forces is inclined more towards the greater force. 3

   (b) If greatest possible resultant of two forces R and S acting at a point is m times the least, show that the angle between them when their resultant is half their sum is $\cos^{-1}\left(-\frac{m^2+2}{2(m^2-1)}\right)$. 3

2. (a) If a force be resolved into two components, one of which is a right angles to the force and equal to it in magnitude, find the direction and magnitude, of the other. 3

   (b) A, B and C are three points on a circle. Forces inversely proportional to AB and BC act along AB and BC respectively. Show that their resultant acts along the tangent to the circle at B. 3

3. (a) Find the resultant of two unlike parallel forces R and S (R > S) acting at two distinct points. 3

   (b) A string is tied to two points at the same level, and a smooth ring of W kg wt which can slide freely along the string, is pulled by a horizontal force of P kg wt. If in the position of equilibrium the portions of string are inclined at 60° and 30° to the horizontal, find the value of P and tension in the string. 3

4. (a) State and prove Lami's Theorem. 3

   (b) A weight W is supported on a smooth plane of inclination β to the horizontal by a force whose line of action makes an angle 2β with

the horizontal. If the pressure on the plane be arithmetic mean of the weight and the force,

show that : $\beta = \frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)$. 3

UNIT–II

5. (a) A 100 kg vertical force is applied at the end B of a tree AB which is fixed in the ground making an angle 60° with the horizontal :

(i) Find the moment of force at B about A.

(ii) What is the magnitude of a horizontal force applied at B which creates the same moment about A ? 3

(b) Three like parallel forces $2R + S$, $4R - S$ and 8N act at the vertices of a triangle. Find R and S if their resultant passes through the centroid of the triangle. 3

6. (a) Show that two coplanar couples of equal and opposite moments are in equilibrium. 3

(b) A light wire, in the form of an arc of a circle subtending an angle β at its centre and having two weights P and Q at its extremities, rests with its convexity downwards upon a horizontal plane. If θ is the inclination to the vertical of the radius to the end at which P is suspended, then show that $\tan\theta = \frac{Q\sin\beta}{P + Q\cos\beta}$ 3

7. (a) ABCD is a rectangle with AB and BC of 'a' and 'b' units respectively. Forces P, P act along AB and CD and forces Q, Q act along AD and CB, where $P > Q$. Prove that the perpendicular distance between the resultant of the forces P, Q at and the resultant of the forces P, Q at C is $\frac{Pb - Pa}{\sqrt{P^2 + Q^2}}$ 3

(b) A uniform ladder of weight W is resting in limiting equilibrium with its one end on a rough horizontal floor and the other end against a smooth vertical wall. Show that the inclination of the ladder to the vertical is twice the angle of friction. 3

8. (a) A heavy rod AB whose centre of gravity divides it into two portions a and b is placed inside a smooth sphere. The rod subtends an angle 2α at the centre. Find inclination of the rod to the vertical and reactions at A and B. 3

(b) If the force which acting parallel to a rough plane of inclination θ to the horizon is just sufficient to draw a weight up be m times the force which will just be on the point of sliding down, show that

$\tan\theta = \frac{m+1}{m-1}\mu$, where μ is the coefficient of friction. 3