MATHEMATICS Paper–II

(Differential Equations – II)

Time Allowed : 3 Hours **Maximum Marks : 30**

Note : Attempt five questions in all, selecting at least two from each unit.

UNIT – I

1. (a) Solve in series the differential equation $y'' - xy' + y = 0$.

   (b) Solve in series the differential equation :

$$(x + x^2)\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} - y = 0$$ 3,3

2. (a) For integral value of n, prove that :
   $J_{-n}(x) = (-1)^n J_n(x)$

   (b) Prove that : $J_n(x) = \frac{1}{2\pi}\int_0^{2\pi} \cos(x\sin\theta - n\theta)\, d\theta$ 3,3

3. (a) If $m \neq n$ then show that : $\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0$

   (b) Show that : $\int_{-1}^{1} x^m P_m(x)\, dx = \frac{2^{m+1}(\lfloor m)^2}{\lfloor 2m+1}$ 3,3

4. (a) Find the general solution of the Lagrange's linear equation :
   $z(xp - yq) = y^2 - x^2$

   (b) Find integral surface of the differential equation $(y-z)p + (z-x)q = x - y$ which passes through the line $y = 2x$, $z = 0$. 3,3

UNIT–II

5. (a) State and prove Linearity property of Laplace transform.

(b) Find Laplace transform of $\frac{\cos\sqrt{t}}{\sqrt{t}}$. 3,3

6. (a) Prove that : $\int_0^\infty e^{-tx^2}\,dx = \frac{1}{2}\sqrt{\frac{x}{t}}$

(b) Evaluate : $L^{-1}\left(\frac{1}{s}\log\left(1+\frac{1}{s^2}\right)\right)$ 3,3

7. (a) Evaluate : $L^{-1}\left(\frac{s^2-2a^2}{s^4+4a^4}\right)$

(b) Apply convolution theorem to evaluate : $L^{-1}\left(\frac{1}{(s+2)^2(s-2)}\right)$ 3,3

8. (a) Solve the initial value problem $X''-3X'+2X=1-e^{2t}$ where $X(0)=1, X'(0)=0$.

(b) Solve $X''+Y''+5x-3Y=0$ and $Y''+3Y-2X=0$ where $X(0)=0$, $Y(0)=0$, $X'(0)2$, $Y'(0)=3$. 3,3