

MATHEMATICS Paper-III

(Dynamics)

Time Allowed : 3 Hours

Maximum Marks : 30

Note : Attempt five questions in all, selecting at least two questions from each unit. Each question will carry 6 marks.

UNIT-I

- (a) A particle moving with uniform acceleration described $\frac{9}{25}$ th of the whole distance in the last second of its motion. If it started from rest, how long was in motion and through what distance did it move if it described 10 m in the 3rd second of its motion.

(b) A ball is dropped from the top of a Tower ' h ' metres high and at the same moment another ball is projected upwards from the bottom. They meet when the upper one has described $\frac{1}{n}$ th of the total distance. Show that their speeds, when they meet are in the ratio 2:

$(n - 2)$ and that the initial velocity of the lower ball is $\sqrt{\frac{1}{2}ngh}$. 3,3

2. (a) Two masses m_1 and m_2 ($m_1 > m_2$) are suspended by a light inextensible string over a smooth pulley. Find the acceleration of masses, tension in the string and pressure on the pulley.
- (b) A body of mass 4 kg is placed at the bottom of a plane inclined at 30° to the horizontal and of length 4 m. It is connected by a string passing over a smooth pulley at the top of the plane to a mass of 3 kg hanging vertically. Find the common acceleration of the masses and the time that elapses before the first particle arrives at the top of the plane. 3,3
3. (a) A particle moves along the x -axis such that its acceleration is :

$$-\frac{k^2}{x^3} \quad (x > 0, k > 0)$$

where x is the co-ordinate of the particle at time t seconds. If R is the co-ordinate of the particle initially, determine x as function of time, given that as $x \rightarrow \infty$, the velocity tends to zero.

- (b) The distance moved by a particle along a straight line is proportional to the square root of the time. Find the acceleration in terms of velocity. 3,3
4. (a) A particle moving with S.H.M. and while moving from one position of rest to other, its distance from the middle point of its path at three consecutive seconds are x_1, x_2, x_3 . Prove that the period of motion

is :
$$\frac{2\pi}{\cos^{-1}\left(\frac{x_1 + x_3}{2x_2}\right)}$$

- (b) Show that the motion of a particle of mass m attached to a vertical elastic string is simple harmonic motion with time period $2\pi\sqrt{\frac{ml}{\lambda}}$.

3,3

UNIT-II

5. (a) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr and $\mu\theta$. Find the path and show that

the acceleration along the perpendicular to the radius vector are

$$\lambda^2 r - \frac{\mu^2 \theta^2}{r} \text{ and } \lambda \theta \left(\lambda + \frac{\mu}{r} \right).$$

- (b) A particle is projected with velocity $2\sqrt{ag}$, so that it just clears two walls of equal height α which are at a distance $2a$ from each other. Show that the latus-rectum of the path is $2a$ and that the time of passing between the walls is $2\sqrt{\frac{a}{g}}$. 3,3

6. (a) A particle describes a circle of radius r making one revolution in n seconds. Prove that the acceleration is directed towards the centre

and is $\frac{4\pi^2 r}{n^2}$.

- (b) An insect crawls at a constant speed u along the spoke of a wheel of radius r , rotating with constant angular velocity ω about its centre. Find actual acceleration of the insect as it reaches the rim of wheel. 3,3

7. (a) A mass of 2 kg falls vertically through 10 metres from rest and is then brought to rest by penetrating $1/5$ metre into a soft bed of sand. Find the average resistance of the sand.

- (b) An engine working at the rate of H units is pulling a train up an incline of 1 in n at a steady rate of v m/sec. If M is the total mass of the train, find the average frictional force. 3,3

8. (a) A bullet of mass 10 gm is fired into a target with velocity 500 m/sec. The mass of the target is 15 kg and is free to move. Find the loss of kinetic energy by impact.

- (b) A sphere of mass 8 kg moving with a velocity of 10 m/sec. collides with another sphere of mass 5 kg moving in the same straight line with velocity 4 m/sec. If the impact is direct and their coefficient of restitution is $3/4$, find their final velocities. 3,3