MATHEMATICS: PAPER II

(Modern Algebra)

Time Allowed: Three Hours

Maximum Marks: 30

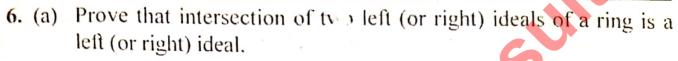
Note: Attempt five questions in all, selecting at least two questions from each Section. All questions carry equal marks.

SECTION-A

- 1. (a) Let G be a finite group and $a \in G$ be an element or order n. Then $a^m = e$ iff n is a divisor of m.
 - (b) If H is a sub-group of G of index 2 in G, then prove that H is normal subgroup of G.
- 2. (a) Let N be a normal sub-group of a group G. Show that G/N is abelian iff for all $x, y \in G$, $xyx^{-1}y^{-1} \in N$.
 - (b) Let H and K be two subgroups of a group G. Show that : $(H \cap K)C = H_C \cap K_C$ for all $C \in G$
- 3. (a) Let G and G' be two groups and $f: G \to G'$ homomorphism of G onto G'. Then prove that $\frac{G}{H} \cong G'$ where H is the kernel of f.
 - (b) If $f: G \to G$ defined by $f(x) = x^3 \ \forall \ x \in G$ is an automorphism, show that G is abelian.
- **4.** (a) If $O(G) = p^2$, where p is prime number then prove that G is an abelian group.
 - (b) Prove that A_4 has no subgroup of order six, A_4 is alternative group of order 4.

SECTION-B

- 5. (a) If in a ring R, $x^3 = x$ for all $x \in R$, then show that R is commutative.
 - (b) Let R be a finite ring without zero divisors and O(R) > 1. Then show that R is a division ring.



- (b) Prove that a commutative ring with unity is simple iff R is a field.
- 7. (a) Show that any ideal of: is maximal iff it is generated by some prime element.
 - (b) Show that the mapping $f: z \to M_2(z)$, defined by : $f(r) = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$ is a homomorphism of rings, find ker f. Is the mapping an isomorphism?
- 8. (a) Let R be a commutative ring with identity and $f(x).g(x) \in R[x]$. Then deg. $[f(x) g(x)] \le \deg f(x) + \deg g(x)$.
 - (b) For any ring R, show that $R[x] \le x \ge \cong R$.