MATHEMATICS Paper–III

(Trigonometry and Matrices)

Time allowed : 3 Hours **Max. Marks : 30**

Note : (i) Attempt **five** questions in all, by selecting at least **two** questions from each unit. (ii) All questions carry equal marks.

UNIT–I

1. (a) Solve the equation $x^5 - 1 = 0$ and show that the sum of n^{th} power of the roots always vanishes unless n be a multiple of 5, n being an integer.

   (b) Expand $\cos^5\theta \sin^7\theta$ in a series of sines of multiples of θ. 3,3

2. (a) If $i^{iii^{\alpha}} = A + iB$ and only principal values are considered, prove that:

(i) $\tan\frac{\pi A}{2} = \frac{B}{A}$

(ii) $A^2 + B^2 = e^{-\pi B}$

(b) If $A + iB = C\tan(x + iy)$, prove that

$$\tan\ 2x = \frac{2CA}{C^2 - A^2 - B^2} \text{ and } \tanh 2y = \frac{2CB}{C^2 + A^2 + B^2}.$$ 3,3

3. (a) If $\sin^{-1}(x + iy) = u + iv$, prove that $\sin^2 u$ and $\cosh^2 v$ are roots of the equation $t^2 - t(1 + x^2 + y^2) + x^2 = 0$.

(b) Sum to n terms the series ;

$$\frac{1}{\sin\theta\sin 2\theta} + \frac{1}{\sin 2\theta\sin 3\theta} + \frac{1}{\sin 3\theta\sin 4\theta} + \ldots\ldots$$ 3,3

4. (a) If $0 < 0\frac{\pi}{4}$, prove that :

$$\log\ \sec\theta = \frac{1}{2}\tan^2\theta - \frac{1}{4}\tan^4\theta + \frac{1}{6}\tan^6\theta \ldots..$$

(b) If $\cos\alpha + 2\cos\beta + 3\cos\gamma = 0 = \sin\alpha + 2\sin\beta + 3\sin\gamma$, prove that : (i)

(i) $\cos 3\alpha + 8\cos 3\beta + 27\cos 3\gamma = 18\cos(\alpha + \beta + \gamma)$

(ii) $\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha + \beta + \gamma)$ 3,3

UNIT–II

5. (a) Show that every Skew-Hermitian matrix A can be expressed uniquely as P-iQ, where P is a real-skew symmetric and Q is a real symmetric matrix.

(b) If $X_1, X_2, \ldots, X_r$ are linearly dependent column vectors of order mx1 and A is any mxm matrix, then show that $AX_1, AX_2, \ldots, AX_r$ linearly dependent. 3.3

6. (a) Find the value of x such that the rank of :

$$\begin{bmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{bmatrix} \text{ is 3.}$$

(b) For the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$, find two non-singular matrices P and Q such that PAQ is in the normal form. 3,3

7. (a) For what value of λ, does the following system of equations have a solution ?

$x+y+z=1, x+2y+4z=\lambda, x+4y+10z=\lambda^2$

(b) Prove that the eigen vectors corresponding to distinct eigen values of a matrix are linearly independent. 3,3

8. (a) Verify Cayley-Hamilton theorem for the following matrix and hence find its inverse (if exists) :

$$\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

(b) Check whether the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 4 & 9 \end{bmatrix}$ is diagonalizable or not.

3,3