MATHEMATICS Paper–III

(Trigonometry and Matrices)

Max. Marks : 30

Time Allowed : 3 Hours

Note : (i) Attempt five questions in all by selecting at least two questions from each Unit.

(ii) All questions carry equal marks.

Unit–I

1. (a) Find the four 4th roots of $1-\sqrt{-3}$. 3

(b) Solve the equation :

$x^9 - x^5 + x^4 - 1 = 0.$ 3

2. (a) Show that root of equation :

$(1+x)^n - (1-x)^n = 0.$

are $i \tan\left(\frac{k\pi}{n}\right)$, $k = 0, 1, 2, 3. \ldots\ldots\ldots\ldots n-1.$ 3

(b) Expand $\cos^7\theta$ in terms of cosine of multiple of θ. 3

3. (a) If $\tan(\theta + i\phi) = \cos\alpha + i\sin\alpha$, show that :

$$\phi = \frac{1}{2}\log\left[\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right].$$ 3

(b) If S_n denote the sum of n terms of the series :

$\sin\theta + \sin 2\theta + \sin 3\theta + \ldots\ldots$ prove that :

$$\lim_{n\to\infty}\frac{1}{n}(S_1 + S_2 + S_3 + \ldots\ldots + S_n) = \frac{1}{2}\cot\left(\frac{x}{2}\right).$$ 3

4. (a) Sum to n terms the series :

$\tan^{-1}\frac{1}{3} - \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots\ldots$ and deduce the sum to infinite terms. 3

(b) If $i^{\alpha+i\beta} = \alpha + i\beta(\alpha,\beta \in \mathbb{R})$, prove that $\alpha^2 + \beta^2 = e^{-(4n+1)\pi\beta}$. 3

Unit–II

5. (a) Prove that a necessary and sufficient condition for a matrix A to be Hermition is that $A^{\Theta} = A$. 3

(b) Define rank of a matrix. Prove that points (x_1, y_1), (x_2, y_2), (x_3, y_3) in a plane are collinear if and only if rank of the matrix :

$$\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix}$$ is less than three. ½+2½

6. (a) Reduce : $A = \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{pmatrix}$ to normal form and hence find its rank. 3

(b) Using elementary operations, find inverse of matrix :

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 0 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$$ 3

7. (a) When a system of linear equations is said to be consistant ? Find the values of λ and μ so that the system of equations :

$$2x - 3y + 5z = 12$$
$$3x + y + \lambda z = \mu$$
$$x - 7y + 8z = 17$$

has (i) a unique solution (ii) infinite solutions (iii) No solution. ½+2½

(b) State and prove Cayley Hamilton theorem. 3

8. (a) Show that the matrix $A\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is not diagonalizable over $\mathbb{R}$, however, A is diagonalizable over $\mathbb{C}$. Find an invertible matrix P over $\mathbb{C}$ such that $P^{-1}AP$ is a diagonal matrix. 1+2

(b) Prove that the modulus of each characteristic root of a unitary matrix is unity. 3