MATHEMATICS Paper–II

(Calculus)

Time allowed : 3 Hours **Max. Marks : 30**

Note : (i) Attempt five questions, selecting at least two questions from each Unit. (ii) Each question will caryy 6 marks.

UNIT–I

1. (a) Between any two distinct real numbers, there exist infinitely many real numbers.

   (b) If $|x-2|<3$ then prove that : $\frac{-18}{7}<\frac{x^2-1}{x+2}<6.$ 3,3

2. (a) Show that $\lim_{x\to 1}\frac{1}{x-1}$ does not exist.

   (b) Evaluate $\lim_{x\to\infty}\frac{\sqrt{3x^2+4x-5}-\sqrt{2x^2+3}}{4x+7}.$ 3,3

3. (a) If f is continuous at $x=a$, then $|f|$ is also continuous at $x=a$.

   (b) Find the value of a and b so that :

   $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}$ exists and is equal to 1. 2,4

4. (a) Evaluate : $\lim_{x\to 0}\frac{(1+x)^{1/x}-e}{x^2}.$

   (b) Evaluate : $\lim_{x\to 0}(\sin x)^{\tan x}.$ 3,3

UNIT-II

5. (a) Differentiate $y = \tanh^{-1}(\sinh x) + \coth^{-1}(x^3) - \text{sech}^{-1}(\cos x)$ w.r.t. x.

(b) Use Cauchy's mean value theorem to evaluate

$$\lim_{x \to 1} \frac{\cos \frac{\pi}{2} x}{\log 1/x}.$$ 3,3

6. (a) Use Mean Value Theorem to prove the following :

$$\pi/6 + \frac{2x-1}{\sqrt{3}} \le \sin^{-1} \le \pi/6 + \frac{2x-1}{\sqrt{1-x^2}} \text{ for } 1/2 \le x < 1.$$

(b) let f be a function defined on [0, x] such that :

(i) $f, f', f'', --, f^{(n-1)}$ are continuous in [0, x]

(ii) $f^{(n)}$ exists in (0, x). Then there exists at least one θ, $0 < \theta < 1$ such that :

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2} f''(0) + \dots + \frac{x^{n-1}}{n-1} f^{n-1}(0) + \frac{x^n}{n} f^n(\theta x)$$

3,3

7. (a) Find n^{th} derivative of : $y = e^{3x} \sin^2 2x$

(b) If $y = (\sin^{-1}x)^2$ then : $(1-x^2) y_{n+2} - (2n+1) x y_{n+1} - n^2 y_n = 0$ 3,3

8. (a) Find $\frac{d}{dx}\left[(x)^{\sinh x} + (\cosh x)^x\right]$

(b) Prove by Rolle's Theorem that equation $4x^3 - 6x^2 + 4x - 1 = 0$ has at least one real root in (0, 1). 3,3