MATHEMATICS Paper–II

(Calculus–I)

Time Allowed : 3 Hours **Max. Marks : 30**

Note : (i) Attempt **five** questions, selecting at least **two** questions from each Unit.

(ii) Each question will carry **6** marks.

Unit–I

1. (a) Solve the inequation : $\dfrac{2}{x-2} < \dfrac{x+2}{x-2} < 2.$

(b) State and prove Archimedian property. Using the property prove that the set of natural numbers N is not bounded above. (3,3)

2. (a) Show that $\lim\limits_{x \to 0} \sin\dfrac{1}{x}$ does not exists.

(b) Evaluate : $\lim\limits_{x \to 1/2} \dfrac{1}{x}\left[\dfrac{1}{x}\right]$, if exists. (3,3)

3. (a) Use intermediate value theorem to show that equation $\sin x - x + 1 = 0$ has a real root.

(b) Evaluate : $\lim_{x \to 0} \frac{x - \sin x}{\tan^3 x}$. (3,3)

4. (a) Evaluate : $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

(b) Discuss the continuity of $f(x) = \begin{cases} \frac{|x| + x}{3}, & x \le 3 \\ \frac{2|x-3|}{x-3}, & x > 3 \end{cases}$ over R.

(3,3)

Unit–II

5. (a) Differentiate $y = x^{\sinh x} + x^{\cosh x}$ w.r.t x.

(b) Let f be a real valued function defined in [a, b] such that (i) f is continuous in [a, b] (ii) f is differentiable in (a, b) (iii) f(a) = f(b), then there exists at least one C∈(a, b) such that $f'(c) = 0$. (2,4)

6. (a) Prove that $\tanh^{-1} x = \frac{1}{2} \log \left(\frac{x+1}{1-x} \right), -1 < x < 1$, and then find its derivative.

(b) Use Cauchy's mean value theorem to evaluate $\lim_{x \to 1} \frac{\frac{\cos \pi x}{2}}{\log \frac{1}{x}}$. (3,3)

7. (a) Use mean value theorem to prove :

$$\frac{x}{1+x} < \log(1+x) < x \text{ for } x > -1, x \neq 0.$$

(b) Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x-2)$. (3,3)

8. (a) State and prove leibnitz's Theorem.

(b) If $y = \frac{\log x}{x}$, prove that $y_n = \frac{(-1)^n \lfloor n}{x^{n+1}} \left[\log x - 1 \frac{-1}{2} \frac{-1}{3} \ldots\ldots \frac{-1}{n} \right]$.

(3,3)