MATHEMATICS Paper–I

(Plane Geometry)

Time allowed : 3 Hours   Max. Marks : 30

Note : Attempt **five** questions, selecting at least **two** questions from each Section.

SECTION–I

1. (a) Find the transformed equation of $11x^2 - 4xy + 14y^2 = 5$ when the axes are rotated through an angle of $\tan^{-1}$.

   (b) Find the joint equation of two straight lines passing through $(1, 2)$ and perpendicular to lines $3x^2 - 8xy + 5y^2 = 0$.   3,3

2. (a) For what value of k the equation : $kx^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represent a pair of straight lines ? Also find the separate equations of lines.

   (b) Find equation of the bisectors of the angle between the lines joining origin to the points of intersection of the curve $x^2 + xy + y^2 + x + 3y + 1 = 0$ and the straight line $x + y + 2 = 0$. 3,3

3. (a) Find the locus of the middle points of the chords of the circle $x^2 + y^2 + 6x + 2y - 10 = 0$ which subtends a right angle at the centre of the circle.

   (b) Find point of intersection of tangents at the points where the line $3x + 4y = 25$ cuts the circle $x^2 + y^2 = 50$. 3,3

4. (a) Find the radical axis and the length of the common chord of the circles $x^2 + y^2 + ax + by + c = 0$ and $x^2 + y^2 + bx + ay + c = 0$.

   (b) Find equation of the circle which belongs to the co-axial system of which the limiting points are (1, –1) and (2, 0) and which passes thought origin. 3,3

SECTION–II

5. (a) Find equation of the common tangents to circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$.

   (b) In the parabola $y^2 = 4ax$, show that the locus of the middle point of the normal PG at P, where G is on the axis, is a parabola. 3,3

6. (a) Prove that the locus of the poles of chords which are normal to the parabola $y^2 = 4ax$ is the curve $y^2 (x + 2a) + 4a^3 = 0$.

   (b) The general equation of a system of parallel chords in parabola $y^2 = 6x$ is $2x + y + k = 0$. What is the equation of corresponding diamerter? 3,3

7. (a) If the eccentric angles of two points on an ellipse differ by $\frac{\pi}{2}$, then show that the tangents to the ellipse at these points intersect on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

(b) Show that the minimum angle between a pair of conjugate diameters of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is $\tan^{-1}\left(\dfrac{2ab}{a^2 - b^2}\right)$. 3,3

8. (a) Find the locus of the mid-points of the chords of $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ which touch the circle $x^2 + y^2 = 1$.

(b) Find the eccentricity of the hyperbola of which $2x - 3y = 0$ and $x = 2y$ is a pair of conjugate diameters. 3,3