MATHEMATICS Paper–I

(Plane Geometry)

Time Allowed : 3 Hours **Max. Marks : 30**

Note : Attempt *five* questions in all, selecting at least *two* questions from each Section.

Section-A

1. (a) Transform $5x^2 - 2xy + 5y^2 + 2x - 10y - 7 = 0$ to rectangular axes through (0, 1) inclined at an angle $\frac{\pi}{4}$ to the original axes.

   (b) Show that $x^2 + (\alpha\sqrt{3}y - 3)x + (3y^2 - 3\sqrt{3}y, -4) = 0$ respresents a pair of straight lines. Also find distance between mean. 3,3

2. (a) Prove that the joint equation of straight lines bisecting the angles between lines :

   $ax^2 + 24xy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

   (b) Find equation of pair of lines joining the origin to the points of intersection of line $y = mx + c$ with the curve $x^2 + y^2 = a^2$. Prove that they are perpendicular if $2c^2 = a^2(1 + m^2)$. 3, 3

3. (a) Find the locus of mid-points of the chords of the circle $x^2 + y^2 = 16$ which touch the circle $(x - 4)^2 + (y - 3)^2 = 36$.

   (b) Find the equation of the circle which passes through the origin and cuts orthogonally each of the circles $x^2 + y^2 - 8x + 12 = 0$ and $x^2 + y^2 - 4x - 6y - 3 = 0$. 3,3

4. (a) The point (2, 1) is a limiting point of a coaxial system of circle of which $x^2 + y^2 - 4y - 3 = 0$ is 9 member. Find the equation of the radical axis and the co-ordinates of the other limiting point.

   (b) Find the equation of circle which passes through the point (2, 0) and touches the straight line $x + 2y - 1 = 0$ at the point $(3, -3)$. 3,3

Section-B

5. (a) Prove that the locus of the middle points of the normal chords of the parabola $y^2 = 4ax$ is :

   $\frac{y^2}{2a} + \frac{4a^2}{y^2} = x - 2a$

(b) Prove that in a parabola the chords of contact of tangents at eight angles passes through focus. 3,3

6. (a) Show that the minimum angle between a pair of conjugate diameter of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\tan^{-1}\left(\frac{2ab}{a^2-b^2}\right)$.

(b) Prove that the locus of the mid-points of the chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which touch the circle on the join of the foci of the ellipse as diameter is :

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2 = a^2e^2\left(\frac{x^2}{a^4}+\frac{y^2}{b^4}\right)$$ 3,3

7. (a) Prove that the pole of $px+my=1$ w. r. t. the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ lies on the ellipse $\frac{x^2}{9a^2}+\frac{y^2}{9b^2}=1$ if $a^2p^2+b^2m^2=9$.

(b) If $y=x$ is a diameter of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and eccentricity of the ellipse is $\frac{1}{\sqrt{3}}$, find the equation of the diameter conjugate to it. 3,3

8. (a) Show that the locus of the mid-points of the chords of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ whose pole lie on the line $x+y-1=0$ is the hyperbola:

$$\frac{x^2}{16}-\frac{y^2}{9}=x+y$$

(b) Find the asymptotes of the hyperbola $xy-x-2y-5=0$. Also find the equation of the conjugate hyperbola. 3,3