# PHYSICS PAPER–A

## (Mechanics–I)

**Time : 3 Hours** **Max. Marks : 22**

**Note :** (i) Attempt *five* questions in all, selecting *two* questions from each of Sections I and II and Section III is compulsory.

(ii) Use of non-programmable scientific calculator is allowed.

(iii) Logarithmic tables may be asked for if needed.

### Section–I

**1.** (a) What are Cartesian and spherical polar coordinates ? How are the coordinates of a point in two systems related to each other ? 3

(b) The motion of a particle can be expressed in terms of the equation $x = 5t - 9, y = 2\cos 3t, z = 2\sin 3t$. Find the magnitude of velocity after 2 seconds. 1

**2.** (a) Prove that velocity of a particle in spherical polar. Coordinate is given by :

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\dot{\phi}\sin\theta\,\hat{\phi}$$ 3

(b) Determine the area of a circle of radius a by using plane polar coordinates. 1

**3.** (a) What is Isotropy of space ? Which law of conservation is explained by it ? Prove this law. 3

(b) A bomb weighing 50 kg explodes into three parts in flight when its velocity is $20\hat{i} + 22\hat{j} + 10\hat{k}$ $ms^{-1}$. Two fragments of the bomb weighing 10 kg and 20 kg are found to have velocities $100\hat{i} + 50\hat{j} + 20\hat{k}$ and $30\hat{i} - 20\hat{j} + 10\hat{k}$ $ms^{-1}$ respectively. Find the velocity of the third fragment. 1

## Section–II

4. (a) Prove that the shape of trajectory of a particle moving under inverse square law force depends on the relationship between the total energy and its angular momentum. 3

   (b) Prove that the centre of mass of two particles divides the line joining the particles in the inverse ratio of their masses. 1

5. (a) Write down Kepler's laws of planetary motion. Prove Kepler's second law of planetary motion. 2½

   (b) If the average distance of mass from the sun is 1·52 times that of the earth from the sun. Find the period of revolution of mass around the sun. 1½

6. What is Rutherford scattering ? Show that differential scattering cross-section for Rutherford scattering by an atomic nucleus is given by :

$$\sigma_{sc}(\theta) = \frac{1}{4}\left(\frac{ze^2}{E}\right)\frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

   where symbols have their usual meaning. 4

## Section–III

*Note* : Attempt any *six* parts, each part carries 1 mark.

7. (i) The Cartesian coordinates of a point are (1, 0, 1). Find the spherical polar coordinates of this point.

   (ii) Prove that :

$$\hat{r} \times \hat{\theta} = \hat{\phi}$$

   (iii) Give *two* examples each of centred and non-central forces.

   (iv) What are the dimensions of the quantity $\frac{L^2}{\mu r^2}$ ?

   (v) In a carbon monoxide molecule (CO), if two atoms are separated by $5{\cdot}6 \times 10^{-10}$ m, locate centre of mass of the system w.r.t. carbon atom.

   (vi) Mention the various forces in nature and which one of them is weakest force ?

   (vii) How is collision between two balls different from a collision between α-particle and a nucleus ? $1 \times 6 = 6$