MATHEMATICS Paper–II

(Calculus)

ime Allowed : 3 Hrs. Maximum Marks : 30

Note : Attempt **five** questions in all, selecting at least **two** questions from ach of the Units I and II.

UNIT–A

1. (a) If a and b are distinct positive real numbers, prove that there exists a natural number n such that na > b. ing this result prove that the set of Natural numbers is not bounded above. 2,1

(b) If $|x-4|<5$, prove that $\frac{-28}{11}<\frac{x^2+1}{x+2}<12$. 3

2. (a) Prove that $\lim_{x\to\frac{-5}{2}} \frac{1}{2x+5}$ does not exist. 3

(b) If $f(x)<g(x)<h(x)$ in some deleted neighbourhood of a and $\lim_{x\to a} f(x) = l = \lim_{x\to a} h(x)$ prove that $\lim_{x\to a} g(x) = l$. 3

3. (a) State Intermediate Value Theorem for a continuous function. Using this theorem, show that the equation $\frac{a}{x-\lambda_1}+\frac{b}{x-\lambda_2}+\frac{c}{x-\lambda_3}=0$, where a, b, c > 0 and $\lambda_1<\lambda_2<\lambda_3$ has two real roots one each in (λ_1, λ_2) and (λ_2, λ_3). 1½,½

(b) Give an example of a function f such that f is continuous nowhere whereas |f| is continuous everywhere. 1

(c) Evaluate $\lim_{x\to 0}\left[\frac{2(\cosh x-1)}{x^2}\right]^{\frac{1}{x^2}}$. 3

4. (a) Let A and B be two nonempty subsets of R and let $C = \{x + y; x \in A$ and $y \in B\}$. If both A and B have supremum, prove that C also has supremum and Sup C = Sup A + Sup B. 3

(b) Evaluate $\lim_{x \to 1} \frac{x^x - x}{x - 1 - \log x}$. 3

UNIT—II

5. (a) State and prove Rolle's theorem. 1.3

(b) If $f(x) = (1-x)^{5/2}$ and

$$f(x) = f(0) + xf'(0) + \frac{x^2}{\lfloor 2} f''(\theta x); 0 < \theta < 1.$$

Find value of-as x tends to 1. 2

6. (a) Using Cauchy's Mean Value Theorem, prove that for $x > 1$ $\lim_{x \to \infty} (x^{1/n} - 1) = \log x$. 2

(b) Find the derivative of $\text{cosech}^{-1} x\ (x > 0)$ w.r.t.x. 1

(c) If $p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that $p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^b}$. 3

7. (a) Using Lagrange's Mean Value Theorem, prove that

$$3 + \frac{1}{28} < (28)^{1/3} < 3 + \frac{1}{27}.$$ 3

(b) If $y = x^2e^x$, show that :

$$y_n = \frac{1}{2} n(n-1) y_2 - n(n-2) y_1 + \frac{1}{2}(n-1)(n-2)y.$$

8. (a) By suitably restricting the domain of cosh x, find the value of $\cosh^{-1} x$. 3

(b) If $f''(x) < 0$ in [0, 1] and $g(x) = f(x) + f(1-x)\ \forall x \in [0, 1]$. Show that g(x) is monotonically increasing in $\left(0, \frac{1}{2}\right)$. 1½

(c) Differentiate w.r.t. x $\tanh^{-1}(\sinh x) + \sinh^{-1}(\text{sech } x)$. 1½