PHYSICS PAPER–A

(Mechanics–I)

Time Allowed : Three Hours **Maximum Marks : 45**

Note : (i) Attempt **five** questions in all, selecting **two** questions from each of Units I and II.

(ii) Unit–III is compulsory.

(iii) Use of non-progammable scientific calculator is allowed.

(iv) Logarithmic tables may be asked for if needed.

UNIT–I

1. (a) Prove that velocity of a particle in spherical polar coordinates is given by :

$$\vec{v} = \dot{r}\,\hat{r} + r\,\dot{\theta}\,\hat{\theta} + r\,\dot{\phi}\sin\theta\,\hat{\phi}$$

where symbols have their usual meanings. 6

(b) The spherical polar coordinates of a point are $(r, \theta, \phi) = (10, 30°, 45°)$; find the Cartesian coordinates of the same point. 3

2. (a) Show that volume element in spherical polar coordinates is given by :

$$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi,$$

where the symbols have their usual meanings. 3

(b) The radius of a ring element cut from a sphere of radius R subtends an angle θ at the centre of the sphere and width of the element

subtends angle $d\theta$ at that point. Show that the solid angle subtended by the ring element at the centre of sphere is

$$d\Omega = 4\pi \sin\frac{\theta}{2}\cos\frac{\theta}{2}d\theta.$$ 3

(c) Find the area of a circule of radius using plane polar coordinates. 3

3. (a) Prove that the law of conservation of energy follows from the homogeneity of time. 6

(b) If a particle of mass m is moving with velocity v, then prove that:

$$\frac{\partial E}{\partial r} = m.a$$

UNIT–II

4. (a) What are central forces ? Show that for a system of two particles moving under the action of central force, the total energy of the system remains conserved. 6

(b) A particle of mass m moves in a circle of radius r under an inverse square force $F = -\frac{k}{r^2}$. Show that total energy of the particle at a point in the circle is $-\frac{k}{2r}$. 3

5. (a) Determine the turning points in the trajectory of a particle moving under a central force. Show, how the total energy is related to the shape. 6

(b) The distance between carbon and oxygen atoms in CO molecule is 1.12 Å Find the centre of mass of CO molecle with respect to O atom. Mass of carbon atom=12 a.m.u. and mass of oxygen atom=16 a.m.u. 3

6. What is Rutheford Scattering ? Show that differential scattering cross-section for Rutherford Scattering by an atomic mucleus is given by

$$\sigma_{sc}(\theta) = \frac{1}{4}\left(\frac{Ze^2}{E}\right)\frac{1}{\sin^4\left(\frac{0}{2}\right)}.$$ 9

UNIT–III

7. Attempt any **six** parts, each part carries $1\frac{1}{2}$ marks :

(i) A particle is moving in X–Y plane. Which component of acceleration in spherical polar coordinates is zero ?

(ii) Define solid angle. Write down its S.I. Unit and dimensional formula.

(iii) Show that spherical polar unit vectors $\hat{r}$ and $\hat{\theta}$ are Orthogonal to each other.

(iv) What happens to the speed of a planet when it comes close to the sun ? Explain.

(v) Mention the various forces in nature and which one of them is weakest force ?

(vi) What are the dimensions of $\frac{L^2}{\mu r^2}$?

(vii) Cartesian coordinates of a point are $\left(1, 0, \sqrt{3}\right)$. Find its spherical polar coordinates (r, θ, ϕ).

$6 \times 1\frac{1}{2} = 9$