MATHEMATICS Paper–I

(Plane Geometry)

Time Allowed : 3 Hrs. **Maximum Marks : 30**

Note : Attempt **five** questions in all, selecting at least **two** questions from each Section.

SECTION–A

1. (a) Show by a suitable translation of axes, that the first degree terms can be removed from the equation $x^2 + 3xy - y^2 + 3x - 7y + 11 = 0$

(b) Find the angle through which the axes may be rotated so that the equation $17x^2 + 12xy + 8y^2 + 13x - 17y + 20 = 0$ may be wanting the product term. Also find the transformed equation. 3,3

2. (a) Show that the equation $x^2 - 5xy + 4y^2 + 4x - y - 5 = 0$ represents a pair of straight lines. Find their point of intersection and angle between them.

(b) Find the equation of the bisectors of the angle between the lines joining origin to the points of intersection of the curve $x^2 + xy + y^2 + x + 3y + 1 = 0$ and the straight line $x + y + z = 0$. 3,3

3. (a) Find the locus of the middle points of the chords of the circle $x^2 + y^2 + 6x + 2y - 10 = 0$ which subtend a right angle at the cube of the circle.

(b) Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$, find the equation of the circles. 3,3

4. (a) Find the equation of the circle which passes through the points (2, 0) and (0, 2) and is orthogonal to the circle $2x^2 + 2y^2 + 5x - 6y + 4 = 0$.

(b) Show that the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ and $x^2 + y^2 + 6x + 2y - 90 = 0$ touch each other internally. Find their point of contact and the equation of the common tangent. 3,3

SECTION–B

5. (a) Prove that the locus of the poles of the tangents to the parabola $y^2 = 4ax$ wrt $y^2 = 4bx$ is parabola $y^2 - \frac{4b^2}{a}x$

(b) Show that tangent at any point P of the parabola bisects the angle between focal chord to P and perpendicular from P to directrix. 3,3

6. (a) Prove that the locus of mid points of the chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which pass through the fixed point (h, k) is $\frac{x^2 - hx}{a^2} + \frac{y^2 - ky}{b^2} = 0$. Also show that it lies on another.

(b) Find the minimum angle between a pair of conjugate diameters of the ellipse $4x^2 + 9y^2 = 36$. 3,3

7. (a) Find the locus middle points of chords of the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which are at a constant distance 'd' from the centre of the hyperbola.

(b) Find the locus of the foot of perpendicular drawn from the centre of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ on any normal. 3,3

8. (a) Find equation of the hyperbola conjgate to the hyperbola $x^2 + 3xy + 2y^2 + 2x + 3y = 0$.

(b) Find the area of the triangle formed by any tangent to the hyperbola $x^2 - y^2 = 4$ and its asymptotes. 3,3