MATHEMATICS Paper–III

(Trigonometry and Matrices)

Time Allowed : 3 Hrs. **Maximum Marks : 30**

Note : (i) Attempt **five** questions in all, by selecting at least **two** questions from each Unit.

(ii) All questions carry equal marks.

UNIT–I

1. (a) Prove that four roots of the equation :

$$16z^4 - 20z^2 + 5 = 0 \text{ are } \pm \sin\frac{\pi}{5}, \pm \sin\frac{2\pi}{5}.$$ 3,3

(b) Define Primitive n^{th} root of unity and prove each Primitive 6th root of unity satisfies $x^2 - x + 1 = 0$. 3,3

2. (a) If $\tan(x + iy) = \sin(p + iq)$. Then prove :
$\coth q \sinh 2y = \cot p \sin 2x$.

(b) If α, β are roots of $z^2 - 2z + 2 = 0$ then prove :

$$\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \frac{\sin n\phi}{\sin^n \phi}, x + 1 = \cot\phi.$$ 3,3

3. (a) If the principal values are considered, prove :

$$\frac{(1+i)^{1-i}}{(1-i)^{1+i}} = \sin(\log 2) + i\cos(\log 2).$$

(b) Prove $1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} \ldots\ldots = \frac{\pi}{2\sqrt{3}}$. 3,3

4. (a) Sum the series to n terms.
$\cot^{-1}(2.1^2) + \cot^{-1}(2.2^2) + \cot^{-1}(2.3^2) + \ldots$

(b) Express $\cos^6\theta \sin^4\theta$ in terms of cosines of multiples of θ. 3,3

UNIT–II

5. (a) Show that every Hermitian matrix A can be uniquely expressed as R+ iS, R, S are real symmetric and real skew-symmetric. Also show $A^\theta A$ is real iff $RS = -SR$.

(b) Under that condition on the scalar $\lambda \in R$, are the vectors $(\lambda, 1, 0)$, $(1, \lambda, 1)$ and $(0, 1, \lambda)$ linearly dependent ? 3,3

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6. (a) Let $A_{3\times3}$ matrix of rank 1 and B be 3×1 matrix. Show rank If [A|B] is either 1 or 2.

(b) Do the following equations have a non-zero solution :

$x-2y-z=0, 2x+5y+2z=0, x+4y+7z=0,$

$x+3y+3z=0.$ 3,3

7. (a) Reduce $A=\begin{bmatrix}1 & 2 & 3 & 5\\ 2 & 3 & 5 & 8\\ 3 & 4 & 7 & 11\end{bmatrix}$ to normal form.

(b) If λ is an eigen value of a non-singular matrix A, prove then $\frac{|A|}{\lambda}$ is an eigen-value of adj A. 3,3

8. (a) State Cayley-Hamilton Theorem. Hence, using it find inverse of martix $A=\begin{bmatrix}2 & 6 & 1\\ 0 & 1 & -6\\ 3 & 4 & -2\end{bmatrix}$.

(b) Define diagonalizable matrix. If $A=\begin{bmatrix}3 & 2 & 0\\ 2 & 0 & 0\\ 1 & 0 & 2\end{bmatrix}$ then diagonalize A.

3,3

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