

MATHEMATICS Paper-III

(Theory of Equations)

Time Allowed : 3 Hours

Maximum Marks : 30

Note : Attempt five questions in all, selecting at least two questions from each Unit. All questions carry equal marks.

UNIT-I

- (a) Prove that complex roots of a real polynomial equation occurs in conjugate pairs. 3

(b) Explain Horner's method of synthetic division to find the quotient and remainder of dividing a polynomial $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, $a_n \neq 0$, where $\alpha \in F$. 3

2. (a) State and prove Vieta's formula. 3
 (b) Solve the equation : $x^4 + 5x^3 - 30x^2 - 30x^2 - 40x + 64 = 0$, given that its roots are equal in G.P. 3
3. (a) Find the condition that the cubic $x^3 + 3px^2 + 3qx + r = 0$ should have its roots in H.P. 2
 (b) If a, b, c are roots of $2x^3 + x^2 + x + 1 = 0$, form an equation whose roots are :

$$\frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a^2}, \frac{1}{c^2} + \frac{1}{a^2} - \frac{1}{b^2}, \frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} \quad 4$$

4. (a) Find the equation whose roots are squared difference of roots of equation $x^3 + 6x^2 + 9x - 4 = 0$ 3
 (b) State Decarte's rule of sign. Use it to find least number of imaginary roots of $2x^7 - x^4 + 4x^3 - 5 = 0$ 1+2

UNIT-II

5. (a) Solve $x^4 - 8x^2 - 24x + 7 = 0$ by Descarte's method. 3
 (b) Apply Fortran's method to solve the equation :
 $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0.$ 3
6. (a) Use Cardon's method to solve :
 $x^3 - 15x^2 - 3x + 847 = 0.$ 3
 (b) Apply Newton's method to solve :
 $x^3 - 5x^2 - 2x + 24 = 0.$ 3
7. (a) For equation $x^3 - 6x^2 - 6x - 14 = 0$, find $G^2 + 4H^3$ and hence discuss the nature of its roots. 3
 (b) Find the interval in which roots lie in the equation :
 $x^4 - 44x^2 + 112x - 384 = 0$ by method of grouping. 3
8. (a) Apply trigonometric method to solve equation $x^3 + 3x + 1 = 0.$ 4
 (b) Reduce the equation $2x^3 - 9x^2 - 13x - 6 = 0$ into which second term is missing. 2