MATHEMATICS Paper-II

(Calculus–II)

Time Allowed : 3 Hours **Maximum Marks : 30**

Note : Attempt *five* questions in all selecting at least *two* questions from each Section.

Section – I

1. (i) Show that origin is the point of inflexion for the curve $y = x^{1/3}$.

(ii) Find the points of inflexion of the curve $y = \frac{x^2+1}{x^2-1}$. Also find the interval where the function is concave upwards and concave downwards. 2,4

2. (i) Find the nature and position of double points of the cure $y(y-6) = x^2(x-2)^3 - 9$.

(ii) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$. 3,3

3. (i) Find all asymptotes of the curve $x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$.

(ii) Find the equation of the cubic curve which has the same asymptotes as the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$, and which pass through the points (0, 0), (2, 0) and (0, 2). 3,3

4. (i) Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$.

(ii) If C_x and C_y be chords of curvature parallel to axes of x and y respectively at any point of the curve $y = ae^{x/a}$, then prove that :

$$C_x^{1/2} + C_y^{1/2} = \frac{1}{2aCx}$$ 3,3

Section – II

5. (i) Evaluate :

$$\int \frac{1}{\sqrt{\cosh 2x + \sinh 2x}} dx$$

(ii) If $I_n = \int_0^{\pi/2} x \sin^n x dx, n > 1, n \in N$, prove that $I_n = \left(\frac{n-1}{n}\right) I_{n-2} + n^{1/2}$.

Hence evaluate I_5. 3,3

6. (i) If $I_{m,n} = \int \sin^m x \cos^n x\,dx$, prove that :

$$I_{m,n} = \frac{\sin^{m+1} x \cos^{n+1} x}{m+1} + \frac{m+n+2}{m+1} I_{m+2,n}$$

Hence evaluate $I_{-2,2}$.

(ii) Use Trapezoidal rule to approximate $\int_0^1 \frac{1}{1+x^2}\,dx$ by taking $n = 4$.

Also find the error. 3,3

7. (i) Evaluate :

$$\lim_{n\to\infty}\left[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots \frac{1}{5n}\right]$$

(ii) Find the area bounded by the curves $y^2 = 8x$ and $x^2 + y^2 = 9$. 3,3

8. (i) Find the length of the arc of parabola $y^2 - 4y + 2x = 0$ which lies in the first quadrant.

(ii) Find the volume of the solid formed by the revolution about x-axis of the loop of the curve $y^2\,(a+x) = x^2(3a-x)$. 3,3