

MATHEMATICS Paper-II

(Calculus – II)

Time Allowed : Three Hours

Maximum Marks : 30

- Note:**
1. Attempt five questions in all, selecting at least two questions from each section.
 2. Each question carries 6 marks.

Section – I

1. (a) Show that the line joining the two points of inflexion of the curve :
 $y^2(x - a) = x^2(x + a)$, $x \neq \pm a$
subtends an angle $\pi/3$ at the origin.
(b) Trace the curve $y^2 = (x + 1)^3$. 3, 3
2. (a) Find the asymptotes of the curve :
 $x^2y + xy^2 + 2x^2 - 2xy - y^2 - 6x - 2y - 2y + 2 = 0$
and show that they cut the curve in at most three points which lie on the straight line $2x - 3y - 4 = 0$.
(b) Determine the position and nature of the double points on the curve:
 $x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$. 3, 3

3. (a) Define circle of curvature. Find the equation of the curve

$$\sqrt{x} + \sqrt{y} = \sqrt{a}.$$

(b) Show that the points of intersection of the curve $xy(x^2 - y^2) - 25x^2 - 9y^2 + 144 = 0$ and its asymptotes lie on ellipse whose eccentricity is $4/5$. 3,3

4. (a) If C_o, C_p denote the lengths of chord of curvatures of the cardioid $r = a(1 + \cos\theta)$ along and perpendicular to the radius vector through any point respectively. Prove that :

$$3(C_o^2 + C_p^2) = 8aC_o.$$

(b) Find the interval in which the curve $y = (x^2 + 4x + 5)e^{-x}$ is concave upwards or downwards. 4,2

Section - II

5. (a) If $\int_0^{\pi/4} \tan^n x dx$, show that, for $n > 1$, $I_n + I_{n-2} = \frac{1}{n-1}$. Hence deduce the value of I_3 .

(b) Evaluate $\int \cosh^{-1} \left(\frac{1+x^2}{1-x^2} \right) dx$. 6

6. (a) Find the length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to any point (x, y) .

(b) Find the volume of the solid obtained by revolving the area included between the curves $y^2 = x^3$ and $x^2 = y^3$ about X-axis. 6

7. (a) Find the surface area of the solid obtained by revolving the curve $y = 2x + 1 + \frac{1}{x^2}$ about x-axis for $1 \leq x \leq 2$.

(b) Use Simpson's rule with $n = 4$ to approximate $\int_{-1}^1 (x^3 + 1) dx$. Also find the error. 3,3

8. (a) Evaluate :

$$\lim_{n \rightarrow \infty} \frac{1}{n^{16}} (1^{15} + 2^{15} + \dots + n^{15}).$$

(b) Derive the reduction formula for $\int x^n \sin(ax) dx$. Hence evaluate

$$\int_0^{\pi/2} x^3 \sin(x) dx.$$

2,4