

Time Allowed : 3 Hours

Maximum Marks : 30

Note : (i) Attempt five questions, selecting at least two questions from each Unit.

(ii) Each question carries 6 marks.

UNIT-I

1. (a) Find the interval in which the curve $y = (x^2 + 4x + 5)e^{-x}$ is concave upwards or downwards.

(b) Show that abscissa of the points of inflexion on the curve

$$x = a - b \cos \theta, y = a\theta - b \sin \theta \text{ is } \frac{a^2 - b^2}{a}. \quad 3,3$$

2. (a) Find all the asymptotes of the curve :

$$(x + y)^2 (x^2 + y^2) - 8(x + y)y^2 + 4x^2 - 3xy - y^2 + 4x + 3 \neq 0.$$

(b) Find asymptotes of $x^4 - 5x^2y^2 + 4y^4 + x^2 - y^2 + x + y + 1 = 0$ and show that asymptotes of the curve cut the curve in at most eight points which lie on a rectangular hyperbola. 3,3

3. (a) Find the position and nature of double points on the curve $x^2 + x^2 + y^2 - x - 4y + 3 = 0$

(b) If $y = (x + 1)^2 (x - 2)$, find the intervals of values of x for which the curve is :

(i) rising

(ii) falling

(iii) concave upwards

(iv) concave downwards

Sketch the graph showing points of inflexion and asymptotes. 3,3

4. (a) Prove that the radius of curvature at any point P on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } \frac{a^2b^2}{p^3}, \text{ where } p \text{ is the length of perpendicular from}$$

the centre of ellipse on the tangent at P.

(b) Find the equation of circle of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the

$$\text{curve } \sqrt{x} + \sqrt{y} = \sqrt{a}.$$

3,3

UNIT-II

5. (a) Evaluate $\int \cos h^{-1} \left(\frac{1+x^2}{1-x^2} \right) dx, |x| < 1.$

(b) Find a reduction formula for :

$\int x^n \sin x dx$ and hence evaluate $\int_0^{\pi/2} x^3 \sin x dx.$ 3, 3

6. (a) Prove that :

$$\frac{2}{\pi} \int_0^{\pi/2} \frac{d\theta}{(1 - e^2 \sin^2 \theta)^{1/2}} = 1 + \frac{1^2}{2^2} e^2 + \frac{1^2, 3^2}{2^2, 4^2} e^4 + \frac{1^2, 3^2, 5^2}{2^2, 4^2, 6^2} e^6 + \dots$$

where $e < 1.$

(b) Find the reduction formula for $I_n = \int e^{ax} \cos^n x dx.$ 3, 3

7. (a) Use Simpson's rule to approximate $\int_0^n \sin x dx,$

taking five ordinates.

(b) Evaluate $\lim_{n \rightarrow \infty} \frac{(\ln n)^{i/n}}{n}.$ 3, 3

8. (a) Find the whole area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}.$

(b) Find the length of the arc of the parabola $y^2 - 4y + 2x = 0$ which lies in the first quadrant. 3, 3