MATHEMATICS Paper–I

(Solid Geometry)

Time Allowed : 3 Hours **Maximum Marks : 30**

Note : Attempt *five* questions, selecting at least *two* questions from each Section.

Section – I

1. (a) Shift the origin to a suitable point so that the equation $2x^2 + 3y^2 + z^2 + xy + zx - x - 10y - 4z + 22 = 0$ is transformed into equation in which the first degree terms are absent.

(b) If $< l_1, m_1, n_1 >$ and $< l_2, m_2, n_2 >$ be the direction cosines of two lines inclined at an angle θ, show that the direction – cosines of the direction bisecting them are :

$$<\left(\frac{l_1 + l_2}{2}\right)\sec\frac{\theta}{2}, \left(\frac{m_1 + m_2}{2}\right)\sec\frac{\theta}{2}, \left(\frac{n_1 + n_2}{2}\right)\sec\frac{\theta}{2}>$$ 3,3

2. (a) Find the equation of the sphere circumscribing the tetrahedron whose faces are $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

(b) Find the locus of the centres of the spheres passing through the fixed point (0, 2, 0) and touching the plane $y = 0$. 3,3

3. (a) Prove that every sphere through the circle $x^2 + y^2 - 2ax + r^2 = 0, z = 0$ cuts orthogonally every sphere through the circle $x^2 + z^2 = r^2$, $y = 0$.

(b) Find the equation of a sphere which belongs to the coaxial system whose limiting points are (1, 2, 0), (2, 2, 0) and which passes through the point (3, -1, 0). 3,3

4. (a) Find the equation of the right circular cylinder described on the circle through the points (2, 2, 0), (0, 2, 0), (0, 0, 2) as the guiding circle.

(b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x-4}{3}=\frac{y}{5}=\frac{z-3}{-4}$ and whose guiding curve is the hyperbola $4x^2-3y^2=5, z=2$. 3,3

Section – II

5. (a) The section of a cone whose vertex is P and guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ by the plane $x=0$ is a rectangular hyperbola. Show that locus of P is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$.

(b) Find the equation of cone with vertex (5, 4, 3) and guiding curve $3x^2+2y^2=6, y+z=0$. 3,3

6. (a) Show that the plane $6x+3y-2z=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines.

(b) Prove that the tangent planes to the cone $lyz+mzx+nxy=0$ are at right angles to the generators of the cone $l^2x^2+m^2y^2+n^2z^2-2mnyz-2nlzx-2lmxy=0$. 3,3

7. (a) Show that $33x^2+13y^2-95z^2-144yz-96zx-48xy=0$ represent a right circular cone whose axis is the line $3x=2y=z$. Find its vertical angle.

(b) Show that the locus of the foot of the perpendicular from the centre of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ to any of its tangent plane is :

$(x^2+y^2+z^2)^2=a^2x^2+b^2y^2+c^2z^2$ 3,3

8. (a) Reduce the equation

$$11x^2+10y^2+6z^2-8yz+4zx-12xy+72x-72y+36z+150=0$$

to the standard form and show that it represents an ellipsoid. Also find the equations of the axes.

(b) If a right circular cone has three mutually perpendicular generators, then show that its vertical angle is $\tan^{-1}\sqrt{2}$. 4,2