MATHEMATICS Paper–I

(Solid Geometry)

Time Allowed : Three Hours **Maximum Marks : 30**

Note : Attempt five questions, selecting at least **two** questions from each section. All questions carry equal marks.

Section – A

1. (a) Shift the origin to a suitable point so that the equation $x^2 + y^2 + z^2 - 4x - 8y + 6z - 4 = 0$ is transformed into an equation in which the first degree terms are absent.

   (b) Transform the equation $13x^2 + 13y^2 + 10z^2 + 8xy - 4yz - 4zx - 144 = 0$ when the axes are rotated to the axes having direction cosines

$$\left\langle -\frac{1}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle, \left\langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3} \right\rangle \text{ and } \left\langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \right\rangle.$$

2. (a) Find the equation of the sphere passing through (1, 0, 0), (0, 1, 0), (0, 0, 1) and whose centre lies on the plane $3x - y + z = 2$.

   (b) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 = 49$, $2x + 3y + 6z = 14$.

3. (a) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z = 3$ and find the point of contact.

   (b) Find the equation of the tangent planes to sphere $x^2 + y^2 + z^2 + 6x - 2z + 1 = 0$ which pass through the lines $x + z - 16 = 0$, $2y - 3z + 30 = 0$.

4. (a) Find the equation of right circular cylinder of radius 3 and having for its axis the line :

$$\frac{x-1}{2} = \frac{y-3}{2} = \frac{5-z}{7}.$$

   (b) Find the equation of cylinder whose generatixes are parallel to the line $\dfrac{x-1}{1} = \dfrac{y+1}{-2} = \dfrac{z-3}{4}$ and whose guiding curve is the parabola $x^2 + 2y = 0, z = 0$.

Section – B

5. (a) Find the equation of the right circular cone whose vertex is at the point (2, 1, –3), semivertical angle 30° and the direction cosines of whose axis are 3 : 4 : –1.

(b) Find the equation of the quadric cone which passes through the three coordinates axes and the three mutually perpendicular lines

$$\frac{x}{1}=\frac{y}{-2}=\frac{z}{3},\ \frac{x}{1}=\frac{y}{-1}=\frac{z}{-1},\ \frac{x}{5}=\frac{y}{4}=\frac{z}{1}.$$

6. (a) Find the equation of the cone whose vector is (2, –3, 1) and whose guiding curve is $4x^2 + y^2 = 1, z = 0$.

(b) Find the equation of the cone circumscribing the sphere $x^2 + y^2 + z^2 + 2x - 2y - 2 = 0$ and having its vertex at (1, 1, 1).

7. (a) Prove that the equation $4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + y = 0$ represents a cone whose vector is (–1, –2, –3).

(b) Find the lines in which the plane $x - 2y - z = 0$ cuts cone $3x^2 + 4y^2 - z^2 = 0$. Find the angle between them.

8. (a) Show that the equation $x^2 + y^2 + z^2 - 6yz - 2zx - 2xy - 6x - 2y - 2z + 2 = 0$ represents a hyperboloid of two sheets.

(b) Reduce the equation

$6y^2 - 18yz - 6zx + 2xy - 9x + 5y - 5z + 2 = 0$ to the standard form.