MATHEMATICS Paper-III

(Theory of Equations)

Time Allowed : 3 Hours **Maximum Marks : 30**

Note : Attempt **five** questions in all selecting at least **two** questions from each unit. **All** questions carry equal marks.

Unit – I

1. (a) Find a polynomial of least degree having $-2, 1, 3$ as its zeros and having value -8 at $x = 2$.

(b) Find g.c.d. of two polynomials $f(x) = x^3 + 6x^2 + 11x + 6$ and $g(x) = x^2 + 7x + 10$. Express the g.c.d as $a(x) f(x) + b(x) g(x)$.

2. (a) Solve the equation $x^4 + 2x^3 - 2x - 1 = 0$ given that it has multiple roots.

(b) Prove that the complex roots of a real polynomial equation occur in conjugate pairs.

3. (a) Solve the equation $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$ given that its roots are in A.P.

(b) Solve the equation $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$ given that two of its roots are equal in magnitude but opposite in sign.

4. (a) Transform the equation $2x^3 - 9x^2 + 13x - 6 = 0$ into one in which second term is missing and hence solve the equation.

(b) If α, β, γ are roots of $2x^3 + x^2 + x + 1 = 0$ form an equation whose roots are

$$\frac{1}{\beta^2} + \frac{1}{\gamma^2} - \frac{1}{\alpha^2}, \frac{1}{\gamma^2} + \frac{1}{\alpha^2} - \frac{1}{\beta^2}, \frac{1}{\alpha^2} + \frac{1}{\beta^2} - \frac{1}{\gamma^2}$$

Unit – II

5. (a) Find the equation whose roots are squared differences of the roots of the equation $x^3 + 6x^2 + 9x + 4 = 0$. Hence show that given equation has a double roots.

   (b) Let $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots\ldots\ldots + a_n x^n$ be a real polynomial of degree n and $a_0 \neq 0$. Let r and s be the number of variations in sign of f(x) and f(–x) respectively. Show that $n - r - s$ is even.

6. (a) Show that the real roots of the equation $x^4 - 10x^3 - 13x^2 + 60x + 65 = 0$ lie between –4 and 12.

   (b) Use Newton's method of divisor to find the integral roots of the equation :

   $3x^4 - 23x^3 + 35x^2 + 31x - 30 = 0$.

7. (a) Use Cardon's method to solve : $x^3 + x^2 - 16x + 20 = 0$.

   (b) For the equation $x^3 - 6x^2 - 6x - 14 = 0$, find $G^2 + 4H^3$ and hence discuss the nature of roots.

8. (a) Solve the biquadratic $x^4 - 6x^3 + 3x^2 + 22x - 6 = 0$ by Descarte's Method.

   (b) Solve by Ferrori's Method, the equation $2x^4 + 6x^3 - 3x^2 + 2 = 0$.